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The Allocation of Scientific Talent

by

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Abstract

I explore the efficiency properties of a decentralized labor market for scientists. I use a model where firms produce science by building labs and hiring researchers in a competitive market. Firms may invest in science to produce new scientific knowledge or to increase their absorptive capacity: the ability to use scientific knowledge produced outside of the firm. In both cases firms underinvest in labs. More interestingly, when firms' investment in science is motivated by absorptive capacity, researchers and labs may be substitutes in the revenue function, even though they are complements in the research production function. This generates a novel form of inefficiency: for any given distribution of labs, the allocation of researchers to firms is non optimal. Subsidies to the investment in labs cannot restore the first best. I show that the existence of scientists' reputation concerns, by preventing the free transfer of surplus between firms and researchers, may affect the allocation of scientists to labs and increase total welfare.

Key words: Knowledge, R&D Productivity, Organization of Scientific Research, Absorptive Capacity, Matching with Investment.

JEL Numbers: D21, H23, L22, O31, O32, O38.

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1 Introduction

How is the allocation of scientific talent determined in a competitive market? Do the best scientists work in the biggest labs, on the most promising research projects, in the location that maximizes their research output? I explore these questions using a model of science production where heterogeneous researchers are hired by heterogeneous firms in order to work in their labs. The ability of each researcher determines the productivity of each lab. The goal of this paper is to describe conditions under which a frictionless job market for scientists generates an inefficient allocation of scientists to labs.

Scientific research carried out by firms produces two distinct outputs, called byCohen and Levinthal (1989) the "two faces of R&D":

- *new science* to be used, for example, in the production of new patents.
- *absorptive capacity* allowing firms to be always up to date with the science produced by other firms and universities. Scientific research provides "a ticket of admission to an information network" (Rosenberg, 1990, p.170). Also, science is difficult: only scientists that are actively engaged in research can read and understand several papers in a timely fashion. In other words, using publicly available science can be costly to firms; in order to lower this cost firms build absorptive capacity by investing in scientific research.

When firms invest in science mainly to produce new science, the allocation of researchers to labs is efficient. In this case, assuming that researchers and labs are complement in the production of science, a firm that invested in a large lab will benefit from a productive researcher more than a firm that invested in a small lab. Hence, the private sector allocation of researcher to labs is Positive Assortative Matching (PAM) assigning productive researchers to large labs. This is also the allocation that maximizes the total amount of science produced for given investment in labs, hence it is the efficient allocation.

On the other hand, when firms invest in science mainly to increase their absorptive capacity, the two inputs in the production of science - labs and scientists - may be substitutes in the firms' revenue function. To understand the intuition, let's consider a simple example: a firm can use science produced by universities and other firms at no cost if and only if this firm achieves a given level of scientific output. To reach this goal, this firm will either invest in a very advanced lab or hire a very productive researcher: it will never invest in a large lab *and* hire a very smart researcher since there is no benefit from producing path-breaking science as opposed to producing just enough science to meet the required threshold. The competitive market allocation of researchers to labs takes the form of a Negative Assortative Matching (NAM) rule: the least productive researcher among the ones that are hired is assigned to the biggest lab in the economy. More in general, the allocation of researchers to labs is NAM whenever firms' marginal benefit from producing science decreases rapidly.[1]

[1] The fact that the marginal benefit from producing science may decrease very rapidly is empirically supported by Gittelman and Kogut (2003), who find that "scientific knowledge and patents are related, but good publications and good patents are not" (ibid, p. 380).

This prediction finds empirical support in the literature on firms size and R&D productivity, showing that small firms are often more productive than large firms in their R&D effort. Assuming that large firms have lower cost of investing in labs,[2] the model predicts that, because of absorptive capacity, there should be a negative relationship between scientists productivity and firm's size. By looking at number of patents produced in firms of different size, several authors document that larger firms are indeed less productive than small firms in their R&D effort.[3] The same type of evidence has been found looking exclusively at scientific research. By collecting data on publicly traded firms, Halperin and Chakrabarti (1987) find that the number of papers produced per dollar of R&D spending is negatively correlated with firms size and with total R&D spending. More directly, Elfenbein, Hamilton, and Zenger (2010) look at the allocation of scientists to firms, and show that productive R&D workers are more likely to work for small firms than for big firms.

When researcher and labs are complements in the science production function, total science produced is maximized under a PAM rule assigning the best researchers to the biggest labs. Therefore, in the allocation of researchers to labs, there is a trade-off between producing science and building absorptive capacity. When firms aim at building absorptive capacity, for any given distribution of labs the private sector minimizes the amount of science produced. Because of the externality in the production of science, the decentralized allocation of researchers to labs is inefficient. This inefficiency is novel and arises in addition to the usual underinvestment in public goods. I show that an appropriate set of taxes/subsidies to the amount of science produced by each firm can solve the inefficiency by inducing the first-best investment and the first-best allocation of researchers to firms. I also show that subsidies to firms' investment alone cannot restore efficiency since they do not affect the job market for researchers.

In the final part of the paper I develop a version of the model where researchers care about reputation, which is built by producing science, and that is not transferable to firms. I show that, if reputation concerns are strong enough, the equilibrium in the private sector may switch from NAM to PAM. Intuitively, researchers may receive most of their compensation in form of reputation rather than as a monetary payment. I also show that the existence reputation concerns may increase total welfare because of its effect on the equilibrium matching pattern. The more general point is that, when there are externalities, welfare may increase by limiting the agents' freedom to bargain over the surplus generated within the match.

1.1 Literature

Given the importance of scientific research as an engine of growth in modern economies, a large body of literature has explored the possible inefficiencies arising in the production of science. Most of these works focused on the public good nature of science and the underinvestment in research (see Arrow, 1962), the tension between basic and applied research (see recently Aghion, Dewatripont, and Stein, 2008), the difficulty

[2] This can be justified in several way. For example, Henderson and Cockburn (1996) show that larger firms tend to have several research programs in different scientific realms. The cost of setting up a new research program in a different scientific field (here, building a lab) is lower for bigger firms, since they can share some costs with other research programs.

[3] See Scherer (1965), Acs and Audretsch (1987), Cohen and Klepper (1996) who review the empirical evidence.

in providing incentives in R&D (see, for example, Aghion and Tirole, 1994). To the best of my knowledge, mine is the first paper looking at the inefficiencies arising in the allocation of scientific talent to different labs.

A closely related literature looked at the allocation of scientific talent to different tasks. In particular, Acemoglu, Aghion, and Zilibotti (2006) develop a growth model where managerial talent can be employed either in innovative activities, or in the adoption of new technology. The main assumption is that talent matters more for innovation than for adoption. Depending on the distance to the scientific frontier, different firms will choose a different strategy and therefore hire a different type of manager. My framework is related, since absorptive capacity and adoption achieve the same goal. However, in my model there is only one task - producing science - that can be performed for different reasons. In addition, I look at the allocation of scientists assuming that all firms share the same motive for science production, and not at the allocation of talent across firms doing research for different reasons.

It has long been observed that, sometimes, firms perform research to improve their ability to use outside knowledge. This idea was first brought forward by Tilton (1971), who analyzes the evolution of the semiconductor industry during the '50s and '60s. Tilton observes that, for firms in this industry, investing in R&D was a form of insurance: it guaranteed to be up to date with the latest scientific breakthrough. The term *absorptive capacity* was introduced by Cohen and Levinthal (1989), who provide both the first theoretical model of this concept and its first empirical test. Other important empirical works are Cockburn and Henderson (1998), Gambardella (1992) and Griffith, Redding, and Reenen (2004). On the theory side, several researchers explored the strategic implications of absorptive capacity (see, for example, Hammerschmidt, 2006, Kamien and Zang, 2000 and Leahy and Neary, 2007), and its implications for economic growth (see Griffith, Redding, and Van Reenen, 2003). My paper contributes to this literature by showing that absorptive capacity has also implications for the allocation of scientists to firms and labs.

The literature on knowledge spillover and geography of innovation (reviewed in Audretsch and Feldman, 2004) shows that, because of local spillovers, the presence of very productive scientists has a positive impact on the productivity of other scientists within the same firm. However, the efficiency properties of the the job market for researchers have not been discussed before, despite headline-grabbing stories about elite scientists leaving one country for another country, or leaving one firm for another firm.[4] With this respect, I show that the allocation of scientists across firms can be inefficient. In some circumstances, there may be an aggregate welfare gain (in addition to a local gain and a local loss) from reallocating scientists from lab to another lab.

Finally, the existing empirical investigations on the allocation of resources to researchers deal exclusively with specific public institutions. For example, Arora, David, and Gambardella (1998) analyze the funding allocation decisions of the Italian CNR (equivalent to the NSF) and show that the reputation (past publication record) is the main explanatory variable. I am not aware of any study looking at the determinants of the allocation of resources to researchers working in the private sector.

[4] For example Liu, M. (2009, November 14). Steal This Scientist. *Newsweek*; or Climbing Mount Publishable: the old scientific powers are starting to lose their grip. (2010, November 11). *The Economist*.

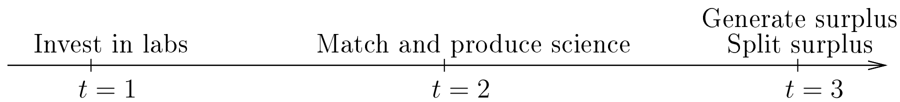


Fig. 1: Timeline

In the next section, I describe the model. In the second section, I characterize the equilibrium for a given distribution of labs. In the third section, I derive the distribution of labs, formally define the equilibrium, and prove its existence. In the fourth section I discuss the normative aspects of the model. I introduce reputation concerns in the fifth section. In the last section I conclude by discussing possible empirical tests, policy implications, and extensions.

2 The Model

The economy is populated by a continuum of firms and a continuum of scientists. Firms differ in their productivity p, continuously distributed over $P = [0, \bar{p}]$, and researchers differ in their ability a, continuously distributed over $A = [0, \bar{a}]$. All agents have the same outside option assumed to be zero. The economy runs for three periods.

2.1 Investing in Labs.

In period $t = 0$ firms build labs. If a firm p sets up labs of size L it bears a cost $c(p, L)$ continuous, positive, with continuous first and second derivative, increasing in L, decreasing in p, with $\frac{\partial^2 c(p,L)}{\partial L^2} \geq 0$ and $c(p, 0) = 0 \, \forall p$.

2.2 Producing Science.

In period $t = 1$, each researcher is hired by one firm and works in the firm's lab. The amount of research produced within each match is:

$$R(a, L) = af(L)$$

where $f(L) \geq 0$, $f'(L) > 0$, and $f''(L) \leq 0$. Note that the two inputs are complements in the research production function. This implies that, for given distribution of labs, the allocation of researchers to labs that maximizes the production of science is Positive Assortative Matching (PAM): the most productive researcher should work in the biggest lab.

The reader should interpret the lab size L as everything that can increase the chance of a scientific discovery for given researcher's ability. This include physical machines (a bigger telescope, a more powerful microscope, a state of the art DNA sequencing machine), as well as the number of technicians and post-docs. The fact that some of these inputs do not require an investment ex-ante but can be purchased after hiring the researcher will turn out to be irrelevant. In the next section I will show that, in equilibrium, firms invest taken as given the researcher allocated to them. This implies that the timing could be reversed with no effect on the equilibrium investment.

Finally, in reality, researchers work in team. This can be incorporated into the model by defining a as the research team's average quality. A previous matching stage determines how researchers form research teams, and how from a distribution of individual ability we can derive the distribution of a. In order to keep the model as simple as possible, I will not pursue this interpretation further.

2.3 The Private Benefit of Research.

At the beginning of the last period ($t = 2$) there is a stock of new science available in the economy. Call its expected commercial value V, and interpret it as the value of all the patents that can be produced out of the available science. The private surplus generated by a match between a researcher and a firm during period $t = 1$ depends on the amount of research carried out in house and the aggregate science V. I assume that the private surplus has an additive form:[5]

$$\Phi(a, L) = V + g(af(L))$$

where $g()$ is continuous and differentiable, $g'() > 0$ and $g''() \leq 0$. The surplus produced is then split between researcher and firm. Finally, V is taken as given by firms and researchers but will be determined endogenously.

Note how firms do not compete with each other on the product market. The reader should imagine a scientific field where many small firms produce patents out of the same scientific base. For example, all firms may belong to the bio-tech sector, some of them developing DNA sequencing machines, some developing drugs, others developing bacteria that can produce bio-fuel out of garbage. Some firms will compete with each other, some will not compete, some other will complement each others. For this reason I abstract from competition issues.

The following two assumptions formally introduce absorptive capacity into the model:

Assumption 1. *It is impossible to understand a new piece of science if no research is carried out in house:* $\lim_{x \to 0} g(x) = -\infty$.

Remember that V represents the new science that will be introduced tomorrow. Under the above assumption, firms need to produce some in-house science today if they want to be active in the market and exploit the new aggregate science V. Note that this does not imply that the science produced in-house

[5] The fact that science enters additively in the private-surplus function will be relevant only when discussing the existence of the general equilibrium. The partial equilibrium analysis will take V as given, and will not be affected by the particular functional form. Furthermore, despite the additive form, in the aggregate there will be a form of complementarity between the stock of science and the return on scientific research. Although each firm's marginal benefit from investing in science does not depend on the stock of science, the number of firms investing will change with V. In particular the higher V, the higher the number of firms producing science, and the higher the total investment in scientific research within a sector.

should be enough to lead to any publication or scientific discovery, neither it implies that all the firms active in the market invest in labs, but it does mean that all the firms active in the market hire a researcher.

Assumption 2. *The marginal benefit of producing science is decreasing rapidly:* $g'''(x) < 0$.

Assumption 2 captures the following consideration. Absorptive capacity implies that firms produce science so that their in-house researchers can be part of the scientific community. Let's say that this is achieved by attending conferences. It follows that a firm will want to produce enough science so that its researcher can attend conferences, but producing even more science provides little extra value. Therefore, the marginal benefit a firm's enjoy from doing research is decreasing rapidly.

Lemma 3. *Under assumptions 1 and 2, from the private sector's point of view the two inputs are always substitutes:*

$$\frac{\partial^2 \Phi(a,L)}{\partial a \partial L} < 0 \, \textit{for every } a, L \in \mathbb{R}^+$$

The proof of proposition 3 is based on the fact that, when both assumptions 1 and 2 hold, the curvature of the cost function $g()$ is given by:

$$-\frac{g''(af(L))}{g'(af(L))} > \frac{\frac{\partial^2 R}{\partial a \partial L}}{\frac{\partial R}{\partial a}\frac{\partial R}{\partial L}} = \frac{1}{af(L)}$$

This curvature implies substitutability.

To have an intuitive grasp about the role played by assumptions 1 and 2, assume for a moment that $g()$ is an isoelastic function. Assumptions 1 and 2 imply that $g()$ is bounded above. This is quite natural whenever firms invest in labs to reduce their cost of using outside science, so that the benefit a firm receives from carrying out research is never above V. This assumptions can also accommodate the case when investing in science has a direct benefit, in the sense that $g()$ can be positive, as long as this benefit has an upper bound.[6]

In what follows, I will assume that absorptive capacity is the main reason why firms perform research in the sense that assumptions 1 and 2 are satisfied. In subsection 3.1 I will address the case where firms invest in scientific research not to increase their absorptive capacity but to produce new science.

2.4 Endogenous Science.

The value of science is taken as given by firms but it is determined endogenously by aggregating all the research carried out in the economy. Call ν the expected commercial value of a unit of research and $h(L)$ the $p.d.f$ of L. The expected value of the stock of science is given by:

$$V = \nu \int m(L) f(L) h(L) dL \tag{1}$$

[6] It is possible to show that boundedness implies local substitutability for large enough $af(L)$. However to have global substitutability one needs to assume 1 and 2: boundedness and assumption 1 or boundedness and assumption 2 are not enough.

where the function $m(L) : \mathbb{R}^+ \to \{A, \emptyset\}$ assigns labs to researchers, with the convention that $m(L) = \emptyset$ represents an unmatched lab. The function $m(L)$ is determined in equilibrium.

3 The Equilibrium for Given Investment in Labs and for Given Aggregate Science.

In this section, I derive the equilibrium arising in period $t = 1$, when firms have already invested in labs. I analyze the problem taking as given the total amount of science produced in the economy V, and the investment made by each firm.

At this stage, firms differ only in the size of the lab they own. Once the distribution of labs is determined the productivity parameter p does not affect the equilibrium anymore. Let's introduce the following notation:

- $x(L) : \mathbb{R}^+ \to \mathbb{R}^+$, the payoff of a firm with lab L.
- $w(a) : A \to \mathbb{R}^+$, the payoff of a researcher with ability a.

Definition 4. For given V, the job market for researchers is in equilibrium if:

- Feasibility: $x(L) + w(m(L)) \leq \Phi(m(L), L)\ \forall L$.
- Stability: $x(L) + w(m(L)) \geq \Phi(m(L'), L)\ \forall L, L'$.

The existence of a unique equilibrium for given V is a standard result in matching theory (see, for example, Kamecke (1992)).

Lemma 5. *Negative assortative matching (NAM) in the job market for researchers: the most productive researchers work in the smallest labs and the least productive researchers work in the biggest labs. Similarly, the most productive researchers work in the smallest firms and the least productive researchers work in the biggest firm.*

Proof. It follows immediately from lemma 3. □

From the firms' point of view, researchers and labs are substitutes. Since the private sector allocates researchers to labs so to maximize their marginal product, it follows that, in equilibrium, the most productive researchers will work in the smallest labs. However, labs and researchers' ability are complements in the research production function. The matching rule maximizing the total stock of science is PAM: the best researcher should work in the biggest lab. Therefore, the private sector, for a given distribution of labs, is minimizing the value of science V. There is a trade-off between maximizing science and maximizing the use of science. Since the private sector only considers the latter, the decentralized equilibrium is inefficient.

Proposition 6. *For given distribution of labs, if ν is high enough, the matching pattern emerging in the private sector is inefficient.*[7]

[7] The equilibrium concept used in this model is called F-core, and the type of externality is called widespread externality. For a theoretical analysis of the inefficiencies of an F-core economy with widespread externalities see Hammond, Kaneko, and Wooders (1989) and Hammond (1995).

Proof. See appendix. □

3.1 Discussion: Beyond Absorptive Capacity.

Suppose now that assumptions 1 and 2 do not hold. The proof of proposition 6 it is based on the fact that, over some range with positive mass of researchers and labs, the social-welfare function is supermodular while the private-surplus function is submodular. This implies that the private sector allocation is inefficient. The reason is that, over that specific range, the equilibrium matching will be NAM, but welfare can be improved by implementing PAM. Therefore, even in situations where assumptions 1 and 2 do not hold, it is possible for the private sector matching pattern to be inefficient, as long as the social welfare function is locally supermodular while the revenue function is locally submodular for some $\{a, L\}$. However, if the function $\Phi(a, L)$ is not globally submodular in a and L, the exact allocation of labs to researchers arising in the market can only be determined numerically.

Suppose now that firms invest in research because they seek to benefit from the knowledge they produce, as opposed to only increasing their absorptive capacity. In this case, the two inputs in the production of science are complements also in the revenue function. Hence the decentralised allocation of scientists to lab is Positive Assortative Matching, assigning the most productive researcher to the biggest lab. This allocation is efficient, since there is no contrast between firms' objective and the production of science. However, Lemma 8 in the next section will show that firms underinvest in labs because they do not fully appropriate the benefit of new science. Therefore, if labs and researchers are global complements, the model collapses back to a standard model of knowledge production where the only source of inefficiency is the firms' underinvesment.

4 The Ex-Ante Equilibrium

Let's introduce some notation:

- $i(p) : P \rightrightarrows \mathbb{R}^+$, the equilibrium investment in labs made by a firm p.
- $\tilde{m}(p) \equiv m(i(p)) : P \rightrightarrows A$, the matching rule on the equilibrium path (for investment performed by some firms) mapping firms to researchers.[8]
- $\tilde{x}(p) \equiv x(i(p)) : P \rightarrow \mathbb{R}^+$, the payoff of firms on the equilibrium path.
- $l(a) \equiv i(\tilde{m}^{-1}(a))$ the lab a researcher of ability a receives in equilibrium.

The definition of equilibrium I use is similar to the one in Cole, Mailath, and Postlewaite (2001). The differences are that, here, only one of the two sides invests.

Definition 7. The quadruple $\{i(.), m(.), x(.), w(.)\}$ constitutes an equilibrium if:

[8] Both $i(p)$ and $\tilde{m}(p)$ are correspondences if all firms are identical and are functions otherwise. $m(L)$ is always a function.

1. The investment is optimal:
$$i(p) = \arg\max_{L \geq 0}\{x(L) - c(p,L)\}$$

2. Ex post, the matching $\{i(.), \tilde{m}(.), \tilde{x}(.), w(.)\}$ is feasible and stable:

   - Feasibility: $\tilde{x}(p) + w(\tilde{m}(p)) \leq \Phi(\tilde{m}(p), i(p))\ \forall p \in P$.[9]
   - Stability: $\tilde{x}(p) + w(\tilde{m}(p')) \geq \Phi(\tilde{m}(p'), i(p))\ \forall p, p' \in P$.

3. For $L \notin \{L : L = i(p) \text{ for some } p \in P\}$ (investments off the equilibrium path):
$$x(L) = \max_a \{\Phi(a, L) - w(a)\}$$

To understand the definition, assume that there is an equilibrium, and consider deviations made by a single firm. Since we are in a large economy, any action this firm may take has no impact on the equilibrium $w(a)$. Therefore, whatever the investment, this firm can match with any researcher a provided that it pays $w(a)$.

Lemma 8. *In equilibrium, for $L \geq 0$:*
$$\frac{\partial x(s,L)}{\partial L} = \frac{\partial \Phi(a,L)}{\partial L}|_{a=m(L)}$$

Proof. From point 3 of the definition of equilibrium. □

Lemma 8 implies that firms' investment solves:
$$\frac{\partial c(s,L)}{\partial L} = \frac{\partial \Phi(a,L)}{\partial L}|_{a=m(L)} \tag{2}$$

In other words, firms maximize surplus taking V and the researchers they will be matched with as given. Since the social planner would take into account the impact of the individual investment on the total stock of science, lemma 8 implies that the investment is inefficient. Finally, note that the matching pattern expected to emerge in the following period affects the investment decisions. It follows, for example, that any policy attempting to change the allocation of researchers to labs will affect the investment. This will turn out to be relevant in section 6.[10] Also, any subsidy to the investment in labs may reduce the underinvestment, but it is unable to affect the inefficiency in the matching between labs and researchers. In the next section I will show that the only way to reach the first best in this economy is using a set of taxes and subsidies to the amount of science produced by each firm.

[9] The general definition of feasibility is more complicated (see Cole et al. (2001)). However, in the cases I consider here it is possible to use this simpler version.
[10] See Gall, Legros, and Newman (2009) for an analysis of this problem in an educational context.

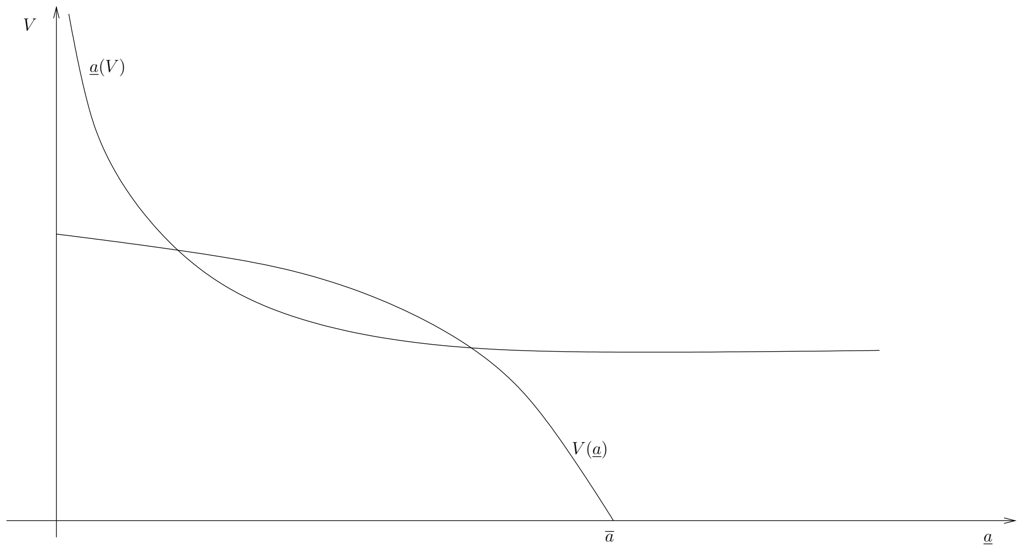


Fig. 2: Equilibrium $\underline{a}$ and V.

Proposition 9. *An equilibrium with zero research always exists. If the commercial value of research ν is high enough, there are also equilibria where a positive amount of science is produced. In these equilibria, researchers belonging to the set $[\underline{a}, \overline{a}]$ match with firms investing $l(a)$, where:*

$$l(a) = max\left\{\left\{L \in \mathbb{R}^+ : \frac{\partial \Phi}{\partial L} = \frac{\partial c}{\partial L}\right\}, 0\right\} \tag{3}$$

$$\underline{a} : \nu \int_{\underline{a}}^{\overline{a}} af(l(a))z(a)da = P(\underline{a}) - g(\underline{a}f(L(\underline{a}))) \tag{4}$$

$$P(\underline{a}) = \int_{\tilde{m}^{-1}(\overline{a})}^{\tilde{m}^{-1}(\underline{a})} \frac{\partial c(s, i(p))}{\partial L} \gamma(p)dp \tag{5}$$

$z(a)$ is the p.d.f. of a, and $\gamma(p)$ is the p.d.f of p.

Proof. See appendix. □

Figure 2 illustrates the case of two positive investment equilibria, given by the intersection of $V(\underline{a})$ and $\underline{a}(V)$, where $V(\underline{a})$ represents the aggregate science produced as a function of the measure of researchers employed, and $\underline{a}(V)$ represents the worst researcher employed in the economy for given aggregate science V. Of the two equilibria represented in figure 2, one can be considered stable (the high V, low $\underline{a}$ one) and the other unstable.

By focusing on the stable equilibrium, it is possible to make a few comparative static exercises. If the value of a discovery ν increases, $V(\underline{a})$ moves upward: more researchers are matched and more research is produced. It is also possible to introduce an exogenous stock of science V^f, science produced, for example, by a foreign country. The graph should be modified by writing on the vertical axes V^h instead of V, and by shifting $\underline{a}(V^h)$ downward: home country is producing more research as well. Obviously, all the comparative statics are reversed if we consider the unstable equilibrium.

5 The First Best

The social welfare generated within each match is:

$$SW(a, L) = \nu af(L) + g(af(L))$$

This function is neither globally supermodular nor globally submodular. It follows that the optimal allocation of researchers to labs can only be derived numerically, and it may involve implementing PAM over some range, and NAM over some other range. Intuitively, the social planner may, over some range, give priority to the production of science, and over some other to the use of science.

However, we know that the social planner problem has a unique solution. This implies that first-best investment in labs and first-best allocation of researchers can be easily implemented if transfers based on the amount of science produced by each firm are feasible.[11]

Proposition 10. *The first best is implementable announcing the following rule: every firm producing some science receives a transfer equal to the value of the science produced by that firm minus V.*

Proof. See appendix. □

Since there is a mass 1 of firms, V is the value of the average amount of science produced. Therefore, firms producing more than the average receive a subsidy, while the others are taxed. However, even if scientific output is observable, it is usually non contractible and, therefore, non taxable. For this reason, the first-best implementation may be impossible to implement.

6 Non-Transferable Utility in The Production of Science: Reputation Concerns.

So far I assumed that the surplus generated within a match can be freely split between scientists and firm. However, the production of science is rid with restrictions to agents' ability to transfer utility, and this may impact the equilibrium allocation arising in the competitive market (see Legros and Newman (2007)). In this section I move to a Non Transferrable Utility (NTU) environment by assuming that part of the surplus generated within a match is in the form of scientists' reputation, and that reputation cannot be transferred back to the firm because of liquidity constraints.[12] I show that NTU has indeed the usual effect on the

[11] See Hammond (1995).

[12] NTU can arise in many other cases. For example, when scientists' effort impact the value of science produced and firms cannot implement incentive schemes depending on the value of output, the particular surplus split agreed ex ante determines total surplus, giving rise to a NTU model.

equilibrium matching: if it is strong enough it will generate a PAM allocation. More interestingly, in this case NTU may be welfare improving since PAM is the the science-maximizing allocation of scientists to labs.

Since the work of Merton (1957), it is well known that researchers care about reputation. Merton calls it the *race for priority*: scientists want to be recognized as the first to discover something. The role of reputation in science has already been explored in the economic literature by Dasgupta and David (1985). Their argument is that, on the one hand, reputation motivates researchers. This is very important because an incentive scheme based exclusively on the quality of scientific output would be very hard to implement. On the other hand, reputation fosters openness. This guarantees the circulation of ideas and generates a faster pace of scientific progress. Here I will show that reputation may affect the production of science in an additional way: by changing the way firms and scientists are matched in equilibrium.

Let's assume that the researchers' utility is:

$$U(a) = w(a) + \rho\Phi(a, L)$$

where $w(a) \geq 0$ is the monetary payment received working for the firm, and $\rho \in (0, 1)$ is the fraction of surplus that accrue to the agent as *reputation*: the utility derived from doing science. Researchers may care about science because their future earning depend on it (through the reputation they build today), or simply because they like science. I assume that the fraction $(1 - \rho)\Phi(a, L)$ of surplus can be split among researchers and firms.

Lemma 11. *For given distribution of labs, if reputation concerns are strong enough*

$$\rho > \frac{\Phi(\overline{a}, \underline{L})}{\Phi(\overline{a}, \overline{L})}$$

the equilibrium matching is PAM. If the value of science ν is large enough, by moving from $\rho = 0$ to some $\rho > \frac{\Phi(\overline{a}, \underline{L})}{\Phi(\overline{a}, \overline{L})}$ social welfare increases.

Proof. In appendix. □

Intuitively, researchers are willing to give up their cash payment in order to produce science.[13] Because of the complementarity between labs and researchers, a productive researcher is always willing to give up more than an unproductive researcher for the right to work in a firm with a given lab. Hence the payoff of both sides (firms and scientists) is increasing in the other side's type. It follows that the equilibrium matching is PAM.

Remember that the social welfare generated within each match is equal to

$$SW(a, L) = \nu a f(L) + g(a f(L))$$

[13] This is consistent with Stern (2004). In his paper "Do Scientists Pay to be Scientists?" the author collects data on job offers received by a sample of biology Ph.D. job market candidates. He finds that firms engaged in science offer wages 25% lower than firms that are not engaged in science.

When reputation concerns change the equilibrium matching from NAM to PAM, for given distribution of labs the effect on the social welfare is twofold. On the one hand, if firms are driven by absorptive capacity, the aggregate cost of using outside science will be higher (i.e. low aggregate $g(af(L))$). On the other hand, the aggregate production of science will increase. This will lead to a welfare improvement as long as the parameter ν (measuring the value of a unit of science) is high enough.

Let's now consider the investment stage. When ρ is large, both benefit and marginal benefit from investing in labs will be low. Hence NTU will depress investment. It is however easy to build cases where the elasticity of the firms investment is low enough so that the overall welfare effect of reputation concerns is positive. For example, setting $c(s, L) = \max\{s, L\}^n$ for n large enough, guarantees that firms will simply set $L = s$ no matter ρ, and the argument made for the case of fixed labs distribution continues to hold.

7 Conclusions

Firms invest in scientific research for several reasons. The one proposed most often in the economic literature is *production*: firms invest in research to increase the stock of scientific knowledge. A second recently-proposed explanation is *absorptive capacity*: using outside science is costly to firms, and this cost is lower if firms produce science in house.

In this paper I build a model where firms invest in labs and hire researchers in order to produce science, and I analyze the competitive allocation of researchers under different motives for science production. I argue that absorptive capacity may lead to an inefficient allocation of researchers to labs. I also show that the presence of non transferability such as reputation may affect this allocation. If the reputation concerns are strong enough, the matching pattern emerging in the private sector is PAM: good researchers work in big labs. It follows that the presence of reputation concerns may increase total welfare.

The model can be tested empirically in several ways. For example, it should be possible to check whether labs and researchers are substitutes in the private sector. Substitutability implies that the increase in revenues following an increase in expenditure in research facilities should be greater in firms with unproductive researchers than in firms with productive researchers. Alternatively, one could check the market allocation of researchers to firms. In this case, however, the test should take into consideration the strength of the reputation concerns. Without reputation, the model predicts NAM. If reputation concerns exist and have the features I derived, we should observe PAM. For example, assuming that old researchers are less sensitive to reputation than young ones, the model predicts that productive young researchers should work in big labs and unproductive young researcher should work in small labs, while productive old researchers should work in small labs and unproductive old researchers should work in big labs.

References

Acemoglu, D., P. Aghion, and F. Zilibotti (2006). Distance to frontier, selection, and economic growth. *Journal of the European Economic Association 4*(1), 37–74.

Acs, Z. and D. Audretsch (1987). Innovation in large and small firms. *Economics Letters 23*(1), 109–112.

Aghion, P., M. Dewatripont, and J. C. Stein (2008). Academic freedom, private-sector focus, and the process of innovation. *RAND Journal of Economics 39*(3), 617–635.

Aghion, P. and J. Tirole (1994). The management of innovation. *The Quarterly Journal of Economics 109*(4), 1185–1209.

Arora, A., P. David, and A. Gambardella (1998). Reputation and competence in publicly funded science: estimating the effects on research group productivity. *Annales d'Economie et de Statistique*, 163–198.

Arrow, K. (1962). Economic Welfare and the Allocation of Resources for Invention. *NBER Chapters*, 609–626.

Audretsch, D. and M. Feldman (2004). Knowledge spillovers and the geography of innovation. *Handbook of regional and urban economics 4*, 2713–2739.

Becker, G. (1973). A Theory of Marriage: Part I. *Journal of Political Economy 81*(4), 813.

Cockburn, I. and R. Henderson (1998). Absorptive Capacity, Coauthoring Behavior, and the Organization of Research in Drug Discovery. *Journal of Industrial Economics 46*(2), 157–182.

Cohen, W. and S. Klepper (1996). A reprise of size and R & D. *The Economic Journal 106*(437), 925–951.

Cohen, W. M. and D. A. Levinthal (1989). Innovation and learning: The two faces of r&d. *Economic Journal 99*(397), 569–96.

Cole, H., G. Mailath, and A. Postlewaite (2001). Efficient Non-Contractible Investments in Large Economies. *Journal of Economic Theory 101*(2), 333–373.

Dasgupta, P. and P. David (1985). Information disclosure and the economics of science and technology. *CEPR Discussion Papers*.

Elfenbein, D., B. Hamilton, and T. Zenger (2010). The small firm effect and the entrepreneurial spawning of scientists and engineers. *Management Science 56*(4), 659–681.

Gall, T., P. Legros, and A. Newman (2009). Mismatch, rematch and investment. *Working Paper*.

Gambardella, A. (1992). Competitive advantages from in-house scientific research: the US pharmaceutical industry in the 1980s. *Research Policy 21*(5), 391–407.

Gittelman, M. and B. Kogut (2003). Does Good Science Lead to Valuable Knowledge? Biotechnology Firms and the Evolutionary Logic of Citation Patterns. *Management Science 49*(4), 366.

Griffith, R., S. Redding, and J. Reenen (2004). Mapping the Two Faces of R&D: Productivity Growth in a Panel of OECD Industries. *Review of Economics and Statistics 86*(4), 883–895.

Griffith, R., S. Redding, and J. Van Reenen (2003). R&d and absorptive capacity: Theory and empirical evidence*. *The Scandinavian Journal of Economics 105*(1), 99–118.

Halperin, M. and A. Chakrabarti (1987). Firm and industry characteristics influencing publications of scientists in large American companies. *R&D Management 17*(3), 167–173.

Hammerschmidt, A. (2006). A strategic investment game with endogenous absorptive capacity. *Department of Economics Working Papers*.

Hammond, P. (1995). Four Characterizations of Constrained Pareto Efficiency in Continuum Economies with Widespread Externalities. *Japanese Economic Review 46*(2), 103–124.

Hammond, P., M. Kaneko, and M. Wooders (1989). Continuum economies with finite coalitions: Core, equilibria, and widespread externalities. *Journal of Economic Theory 49*, 113–134.

Henderson, R. and I. Cockburn (1996). Scale, scope, and spillovers: the determinants of research productivity in drug discovery. *The RAND Journal of Economics 27*(1), 32–59.

Kamecke, U. (1992). On the Uniqueness of the Solution to a Large Linear Assignement Problem. *Journal of Mathematical Economics 21*, 509–21.

Kamien, M. and I. Zang (2000). Meet me halfway: research joint ventures and absorptive capacity. *International Journal of Industrial Organization 18*(7), 995–1012.

Kaneko, M. and M. Wooders (1996). The nonemptiness of the f-core of a game without side payments. *International Journal of Game Theory 25*(2), 245–258.

Leahy, D. and J. Neary (2007). Absorptive capacity, R&D spillovers, and public policy. *International Journal of Industrial Organization 25*(5), 1089–1108.

Legros, P. and A. F. Newman (2002). Monotone matching in perfect and imperfect worlds. *Review of Economic Studies 69*(4), 925–42.

Legros, P. and A. F. Newman (2007). Beauty Is a Beast, Frog Is a Prince: Assortative Matching with Nontransferabilities. *Econometrica 75*(4), 1073–1102.

Merton, R. (1957). Priorities in scientific discovery: a chapter in the sociology of science. *American Sociological Review 22*(6), 635–659.

Merton, R. (1979). *The sociology of science: Theoretical and empirical investigations.* University of Chicago Press.

Nelson, R. (1959). The Simple Economics of Basic Scientific Research. *The Journal of Political Economy 67*(3), 297.

Rosenberg, N. (1990). Why do firms do basic research (with their own money)? *Research Policy 19*(2), 165–174.

Scherer, F. (1965). Firm size, market structure, opportunity, and the output of patented inventions. *The American Economic Review 55*(5), 1097–1125.

Stern, S. (2004). Do scientists pay to be scientists? *Management Science 50*(6), 835–853.

Tilton, J. (1971). International Diffusion of Technology: The Case of Semiconductors. *The Brookings Institution, Washington, DC*.

Zenger, T. (1994). Explaining organizational diseconomies of scale in R&D: Agency problems and the allocation of engineering talent, ideas, and effort by firm size. *Management Science 40*(6), 708–729.

Zucker, L., M. Darby, and M. Brewer (1998). Intellectual human capital and the birth of US biotechnology enterprises. *The American Economic Review 88*(1), 290–306.

A Appendix

Proof of Proposition 3.

It is straightforward to check that substitutability at a given $\hat{a}, \hat{L}$ is equivalent to:

$$\frac{g''(\hat{a}f(\hat{L}))\hat{a}f(\hat{L})}{-g'(\hat{a}f(\hat{L}))} > 0$$

the proof of the proposition requires two steps:

1. *Show that under assumption 2* $r(x) \equiv \left[\frac{g''(x)x}{-g'(x)}\right]$ *is increasing in* x.
   Compute $r'(x)$
   $$r'(x) = \frac{g''(x)}{-g'(x)} + \frac{g'''(x)x}{-g'(x)} + \frac{g''(x)x}{(g'(x))^2}$$
   that is increasing if $g'''(x) > 0$.

2. *Show that under assumption 1,* $lim_{x \to 0} r(x) \geq 1$
   suppose not: $\exists \epsilon > 0$ arbitrarily close to zero such that $g''(\epsilon)\epsilon < -g'(\epsilon)$. Take an arbitrary $\sigma > 0$ and

define:

$$K_{\epsilon,\sigma}(x) \equiv a_{\epsilon,\sigma}\left(\frac{x^{1-\sigma}}{1-\sigma}\right) + b_{\epsilon,\sigma}$$

where $a_{\epsilon,\sigma}$ and $b_{\epsilon,\sigma}$ are such that:

$$K_{\epsilon,\sigma}(\epsilon) \equiv a_{\epsilon,\sigma}\left(\frac{\epsilon^{1-\sigma}}{1-\sigma}\right) + b_{\epsilon,\sigma} = g(\epsilon)$$

$$K'_{\epsilon,\sigma}(\epsilon) \equiv a_{\epsilon,\sigma}\epsilon^{-\sigma} = g'(\epsilon)$$

since we assumed that $g''(\epsilon)\epsilon < -g'(\epsilon)$, it follows that:

$$g''(\epsilon) < \frac{-g'(\epsilon)}{\epsilon} = a_{\epsilon,\sigma}\epsilon^{-\sigma-1}$$

because of the strict inequality, it is always possible to take a $\sigma < 1$, arbitrarily close to one, such that:

$$g''(\epsilon) < a_{\epsilon,\sigma}\sigma\epsilon^{-\sigma-1} = K''_{\epsilon,\sigma}(\epsilon)$$

this implies that, in a neighborhood of ϵ, $g(x) < K_{\epsilon,\sigma}(x)$. Finally, note that $x = 0$ is in a neighborhood of ϵ and at the same time $K_{\epsilon,\sigma}(0)$ is well defined for $\sigma < 1$. Therefore $g(0)$ is well defined and finite. This is a contradiction.

Point 2 alone implies that the inputs are substitutes for small enough $af(L)$. Point 1 and point 2 imply that the two inputs are always substitutes.

Proof of Proposition 6.

The social welfare generated within each match is equal to:

$$SW(a,L) = \nu af(L) + g(af(L))$$

one obvious difference between the first best allocation and the private sector allocation is in who is matched. In the private sector, researchers and labs are matched if $V \geq -g(af(L))$. Note that V is determined endogenously, and that there are multiple equilibria. However, the private sector condition for being matched is, in general, different than the social optimal one.

Going back to the matching pattern, note that NAM is inefficient only under some conditions on ν. To see this, imagine that the economy is so unproductive (low ν) that both from the social point of view and from the private point of view, nobody should be matched. In this case any matching pattern will lead to the same welfare (zero) so that NAM is trivially efficient.

It is easy to show that $SW_{12} > 0$ if:

$$\nu + g'(x) + xg''(x) > 0$$

Given this, we can be in one out of three possible situations. The first one is illustrated in figure 3a. In this case there is no complementarity in the relevant range of the social welfare function and NAM is efficient. Imagine now to increase ν. The area of complementarity expands, and eventually we reach the situation illustrated in figure 3b. In this case, it is possible for the social planner to reallocate some researchers and some labs in order to have an area of PAM. However, this leaves some unmatched agents, that should be re-matched somehow. Whether this deviation increases social welfare or not is left to be determined in future works. If ν is even higher, eventually the economy will reach the situation depicted in figure 3c. In this case it is possible to rematch researchers between a^1 and a^2 with labs from L^1 and L^2 according to PAM and increase the social welfare.

Proof of Proposition 9.

For the first part, note that if firms expect $V = 0$, they have no reason to invest in research. Therefore, the total science produced will be zero.

Consider an equilibrium with positive investment. In general, if all the researchers and all the entrepreneurs in the economy were matched, the worst member of each group could enjoy a strictly positive payoff. In our case, since the worst researcher in the economy is $a = 0$ and $\lim_{a\to 0} \Phi(a, L) = -\infty$, on both sides there is always someone that is not matched. Consider the match between the firm that invested the most and the worst researcher. The researcher receive a payoff equal to zero, while the firm receives

$$\tilde{x}(\overline{p}) = \int_{\tilde{m}^{-1}(\overline{a})}^{\tilde{m}^{-1}(\underline{a})} \frac{\partial \Phi(\tilde{m}(p), i(p))}{\partial L} \gamma(p) dp = \int_{\tilde{m}^{-1}(\overline{a})}^{\tilde{m}^{-1}(\underline{a})} \frac{\partial c(p, i(p))}{\partial L} \gamma(p) dp \equiv P(\underline{a})$$

where $\gamma(p)$ is the p.d.f. of p, and the second equality follows from lemma 8 and equation 2. In other words, the payoff received by the most productive firm depends on the productivity of the worst researcher matched. The equilibrium $\underline{a}$ and V are the solutions to:

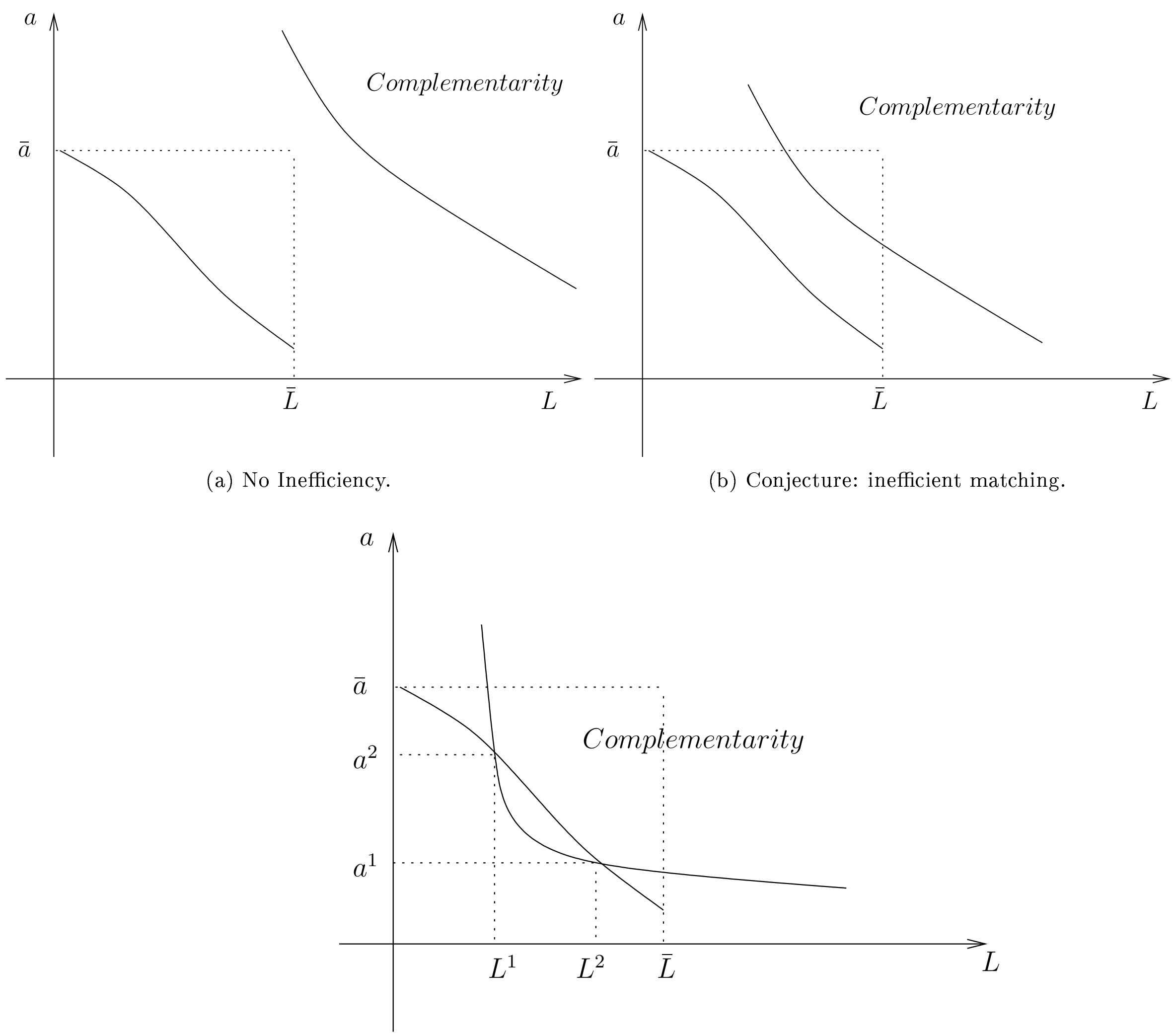
$$\underline{a} = \{a : \Phi(a, l(a)) = P(a)\} \tag{6}$$

and:

$$V = \nu \int_{\underline{a}}^{\overline{a}} af(L(a))z(a)da \tag{7}$$

The equilibrium with positive investment exists if there is a $\{\underline{a}, V\}$ solution to equations 6 and 7.

Note that equation 7 has a finite value at $\underline{a} = 0$, is equal to zero at $\underline{a} = \overline{a}$, and is strictly decreasing.



(a) No Inefficiency.

(b) Conjecture: inefficient matching.

(c) Inefficient matching.

Fig. 3: Complementarity range and matching function.

Finally, equation 6 can be rewritten as:

$$V = P(\underline{a}) - g(\underline{a}f(L(\underline{a}))) \tag{8}$$

Because of assumption 1, if $\underline{a} \to 0$ the solution to 8 diverges to infinity, has finite values for $\underline{a} \in (o, \overline{a}]$, and is continuous. Therefore, if ν is high enough, equations 6 and 7 will cross.

Proof of Proposition 10.

The social welfare generated in each match is equal to:

$$SW(a, L) = \nu af(L) - g(af(L))$$

the private surplus is:

$$\Phi(a, L) = V - g(af(L))$$

clearly, a transfer like the one described transforms the private surplus into the social welfare function. Finally, because of lemma 8, when firms invest they equate marginal cost to marginal benefit. In this case, it implies that firms' investment is efficient.

Proof of Lemma 11.

I will say that NTU is binding whenever a scientist receives no cash payments and is rewarded exclusively in reputation. It is easy to see that, when NTU is binding, the equilibrium must be (local) PAM, since both sides' payoffs are (locally) increasing in the other side's type. Also, if NTU is not binding, the equilibrium cannot be local PAM.

Assume that NTU is binding all the way. The best researcher could match with the smallest lab, since the firm with the smallest lab is the one that can offer the largest cash payment. This is not a profitable deviation as long as:

$$\rho\Phi(\overline{a}, \overline{L})) > \Phi(\overline{a}, \underline{L})$$