DEPARTMENT OF ECONOMICS
WORKING PAPERS

economics.ceu.hu

Money, Banking and Interest Rates: Monetary Policy Regimes with Markov-Switching VECM Evidence

by

Giulia Ghiani[1], Max Gillman[2], Michal Kejak[3]

2014/3

[1] Milan Polytechnic
[2] Economics Department, Central European University; University of Missouri, gillmanm@umsl.edu
[3] CERGE-EI Prague

Abstract

The paper sets out theory and evidence for the equilibrium determination of the nominal interest rate. We test the cash-in-advance economy using US postwar data and find cointegration of the interest rate, inflation, unemployment and the money supply, using either M2 or M1 monetary aggregates, and the Federal Funds rate or the three month Treasury bill rate. Results are consistent both with a persistent monetary liquidity effect in the cointegrating vector coefficients and also a long run quantity theoretic relation. We identify three Markov-switching regimes similar to NBER contractions, expansions, and the "unconventional" period. Dropping money indicates model misspecification.

Keywords*:* Euler equation, money supply, non-stationarity, cointegration, Markov-Switching VECM.

JEL: C32, E40, E52

Acknowledgement:

The authors are grateful to Guglielmo Maria Caporale, Lucio Sarno, Pierre Siklos, Tao Zha and participants at 2014 MMF Conference, 2014 CEF Conference, and 14th Annual Missouri Economics Conference.

1 Introduction

We present a general theory and US postwar evidence for the cointegration of the nominal interest rate with the money supply growth rate, the inflation rate and unemployment rate. Theoretically, the model includes market segmentation in the supply of exchange credit through a micro-based banking production functions that endogenizes the velocity of money (as in Benk et al. 2005, 2008, 2010). This leads directly to two aspects missing from related cash-only models. First the inflation coefficient in the interest rate pricing condition has inviolable features as characterized in the literature as the Taylor principle in terms of it being greater than or equal to one. This holds even though the theory includes a reaction function only implicitly in terms of the government's stochastic money supply process both in the long run and across regimes as it reacts to shocks (Davies et al., 2012). Second the model allows a liquidity effect of money on the interest rate through the bank intermediation approach to market segmentation as an alternative approach to using market segmentation in terms of access to the bond market (Alvarez et al., 2001, Alvarez and Lippi, 2014).

Both inflation effects and Alvarez and Lippi (2014) type persistent liquidity effects find support in the empirical vector error correction model (VECM) results in such a way that the quantity theoretic relation between money supply growth and inflation holds in the long run. The long run results are in terms of the nature of the error term of the cointegrating vector error term and in terms of the cointegrating vector coefficients. The error term strongly correlates with the difference between the money supply growth rate and the inflation rate, consistent with the model's long term determination of the inflation rate by the money supply growth rate. This means the unexplained part of the vector is when the money supply growth rate deviates from the inflation rate. Consistent with the model, the cointegrating vector coefficients show a positive inflation effect on the nominal interest rate and a negative one effect from the money supply growth rate, indicative of how a persistent liquidity effect of continued money supply injections lower the nominal interest rate even as the money supply growth rate minus the inflation rate is essentially the error term of the cointegrating vector.

Motivated by Sims and Zha (2002, 2006), we employ a Markov switching VECM (MS-VECM) to identify three distinct regimes that are alternative to chronological Fed chairmen, but also have a sense of active and passive monetary policy that has become a popular characterization of regimes (Leeper and Zha, 2003). Monetary policy dynamics across regimes differs as to whether inflation or the money supply growth rate is significant, with the former interpreted here as passive monetary regimes and the later as more active ones. The three regimes are contractions, expansions, and "unconventional" mostly negative real interest rate periods. On the "real" side, during the contraction and expansion regimes the past unemployment rate changes negatively explain current short run nominal interest rate changes, as is consistent with an interpretation of the real interest rate being driven lower by rising unemployment and this getting reflected in lower nominal rates. On, the monetary side, contractions appear more active in that past money supply growth rate changes negatively explain current short run nominal interest rate changes, while inflation changes show insignificance. Expansions appear more passive in that past inflation rate changes explain current interest rate changes, but not past money supply growth rate changes. The only dynamic factor present in all three regimes is a type of interest rate "smoothing" in terms of the current interest rate changes depending significantly on past period interest rate changes. The "unconventional" regimes shows that the nominal interest rate depends significantly only on these past changes in nominal interest rates, along with a significant long run adjustment term, making it more of a drifting regime rather than either an active or passive monetary regime in the sense used for expansions and contractions.

Our dynamic monetary model with an exogenous, stochastic money supply growth rate has a stationary balanced growth path value of the money supply growth rate that achieves a long run Lucas and Stokey (1983) type of inflation rate targeting as in Alvarez and Lippi (2014). The endogenous velocity gives rise to key elements of the interest rate Euler equation which is then combined with the cash-in-advance constraint to produce the equilibrium asset pricing relation used in estimation. Going from this equilibrium to estimation we assume that the expected future nominal interest rates are given "naively" by the current money supply growth rate, and we proxy the expected growth in leisure by the unemployment rate. This results in an estimation model that is "observationally equivalent" to some versions of the Taylor (1993) rule that have been estimated, except that it is extended to include the money supply growth rate (see McCallum, 1999, for a related approach).

The regimes are related to the three Markov-switching regimes of Gillman et al. (2014), which are contraction, expansion and "lost decades". This forms an alternative foundation for the monetary regime approach, and so builds upon the money supply based approach of Sims and Zha (2006), as well as the Leeper and Zha (2003) and Bianchi and Ilut (2013) chronological Federal Reserve chairmen approach to regimes. Our cash-in-advance assumed unit root I(1) in the money supply stock, in the face of I(1) evidence for the money supply growth rate, requires the implicit presumption of theoretical "boundedness" of the money supply process, a qualification that is a mainstay of recent monetary regime work (e.g. Davig and Leeper, 2007). Given this, the estimated cointegrating vector and its regime-dependent transition dynamics can be interpreted as a de facto monetary policy rule (see McCallum 1990, Leeper and Roush, 2003, Sims and Zha, 2006, Woodford, 2008, Thornton, 2014). It avoids the problem of unbalanced regressions (the inclusion of stationary series along with I(1) series) that Siklos and Wohar (2005) find in many Taylor rule estimations, in that we use all I(1) series. The Markov-switching transition probabilities relate to the literature on a stochastic movement in the inflation targets; see Ireland (2007), Erceg and Levin (2003), Smets and Wouters (2007), Cogley and Sbordone (2005), Gavin et al. (2005), Roberts (2006), and Salemi (2006).

Section 2 presents the cash-in-advance economy. Section 3 provides the methodological framework of the empirical analysis and the cointegration analysis, along with estimation of a three-state MS-VECM and a Rolling Trace test for robustness. Section 4 focuses on the interpretation of the cointegrating vector, its error term, and its robustness using alternative money and interest rate series, while Section 5 concludes. Appendices include unit root tests (Appendix A), data analysis (Appendix B), MS regime and lag selection (Appendix C), an alternative two state MS analysis (Appendix D), estimation of the baseline VECM model using M1 instead of M2 (Appendix E) and the baseline estimation using the 3-Month Treasury Bill rate instead of the Federal Fund rate (Appendix F).

2 Representative Agent Exchange Economy

The model is from Benk et al. (2008, 2010). It is a cash-in-advance monetary economy with costly exchange credit provided by the financial intermediation sector so as to endogenize the velocity of money. In addition this includes endogenous growth. Building on Davies et al. (2013), the model implies that there exists a log-linearized Euler condition that is similar in form to a "Taylor rule", but it is merely an asset pricing equilibrium condition rather than a reaction function. Using endogenous growth causes the target values of this log-linearized condition to be equal to the balanced growth path (BGP) equilibrium values of the variables, so that there is a Lucas and Stokey (1983) type of long run inflation targeting determined mainly by the mean BGP money supply growth rate. [1]

First consider the special cash-only case of the Benk et al. (2008, 2010) economy that omits the exchange credit option as a substitute for cash. As in Schabert (2003), we show next that the parameter on the log-linearized Euler equations is one, instead of being greater than one, as is the case when the model is mimicking the "Taylor principle". Spelling out this model, let there be shocks to the goods sector productivity, z_t, and to the money supply, ζ_t. Shocks occur at the beginning of the period, are observed by the consumer before the decision making process commences, and follow a vector first-order autoregressive process, whereby

$$S_t = \Phi_S S_{t-1} + \varepsilon_{st}, \tag{1}$$

and the shock vector is $S_t = [z_t \ \zeta_t]'$, the autocorrelation matrix is $\Phi_S = diag\left\{\varphi_z, \varphi_\zeta\right\}$, $\varphi_z, \varphi_\zeta \in [0,1]$ are the autocorrelation parameters, and the shock innovations are $\varepsilon_{St} = [\epsilon_{zt} \ \epsilon_{\zeta t}]' \sim N(\mathbf{0}, \mathbf{\Sigma})$. The general structure of the second-order moments is assumed to be given by the variance-covariance matrix $\mathbf{\Sigma}$ such that there may be correlation between the shocks.

[1] which enables "medium term cyclic" inflation tax effects on output growth.

2.1 Cash Only Economy

The special cash-only economy with such shocks can be thought of as a Cooley and Hansen (1989) economy extended with endogenous growth as in Gomme (1993). The representative consumer has current period utility U_t from consumption of goods, c_t, and leisure, x_t, as given by $U_t = \frac{(c_t x_t^{\psi})^{1-\sigma}}{1-\sigma}$, with time discount factor $\beta \in (0, 1)$, and with $\psi > 0$ and $\sigma > 0$. Output of goods, y_t, and investment in human capital are produced with physical capital and human capital each in Cobb-Douglas fashion. Let s_{Gt} and s_{Ht} denote the fractions of physical capital k_t that the agent uses in goods production and human capital investment whereby $s_{Gt} + s_{Ht} = 1$. The agent allocates time fractionally to leisure, x_t, labor in goods production, l_{Gt}, and time spent investing in the stock of human capital, $l_{Ht} : l_{Gt} + l_{Ht} + x_t = 1$. Output of goods can be converted into physical capital, k_t, without cost and is thus divided between consumption goods and physical capital investment, denoted by i_t. With a fixed rate of capital depreciation $\delta_k \in (0, 1)$, the capital stock used for production in the next period is $k_{t+1} = (1 - \delta_k)k_t + i_t = (1 - \delta_k)k_t + y_t - c_t$. Human capital investment is produced using physical capital $s_{Ht}k_t$ and human capital $l_{Ht}h_t$, where h_t denotes the stock of human capital at time t, the l_{Ht} time allocation gives the fraction of human capital devoted to producing new human capital, and where $A_H > 0$, $\eta \in [0, 1]$ and $\delta_h \in (0, 1)$, and $h_{t+1} = (1 - \delta_h)h_t + A_H(s_{Ht}k_t)^{1-\eta}(l_{Ht}h_t)^{\eta}$.

With w_t and r_t denoting the real wage and real interest rate, the consumer receives nominal income of wages and rents, $P_t w_t l_{Gt} h_t$ and $P_t r_t s_{Gt} k_t$, and a nominal transfer from the government, T_t. With other expenditures on goods, of $P_t c_t$, and physical capital investment, $P_t k_{t+1} - P_t(1 - \delta_k)k_t$, and investment in cash for purchases, of $M_{t+1} - M_t$, and in nominal bonds, $B_{t+1} - B_t(R_t)$, where R_t is the gross nominal interest rate, the consumer's budget constraint is:

$$
\begin{aligned}
& P_t w_t \left(l_{Gt} + l_{Ft}\right) h_t + P_t r_t s_{Gt} k_t + T_t \\
\geq \; & P_t c_t + P_t k_{t+1} - P_t(1 - \delta_k)k_t + M_{t+1} - M_t + B_{t+1} - B_t(R_t).
\end{aligned}
\tag{2}
$$

The standard money-only cash-in-advance (CIA) constraint is

$$
M_t + T_t \geq P_t c_t. \tag{3}
$$

Given k_0, h_0, M_0 and the evolution of M_t $(t \geq 0)$ as given by the exogenous monetary policy in equation (4) below, the consumer maximizes the lifetime discounted utility flow subject to the budget and exchange constraints (2) and (3).

The firm maximizes profit given by $y_t - w_t l_{Gt} h_t - r_t s_{Gt} k_t$, subject to a standard Cobb-Douglas production function in physical and human capital: $y_t = A_G e^{z_t}(s_{Gt}k_t)^{1-\alpha}(l_{Gt}h_t)^{\alpha}$. The first order conditions for the firm's problem yield the standard expressions for the wage rate and the rental rate of capital: $w_t = \alpha A_G e^{z_t} \left(\frac{s_{Gt}k_t}{l_{Gt}h_t}\right)^{1-\alpha}$, $r_t = (1 - \alpha) A_G e^{z_t} \left(\frac{s_{Gt}k_t}{l_{Gt}h_t}\right)^{-\alpha}$. It is assumed that government policy includes sequences of nominal transfers as given by:

$$T_t = \Theta_t M_t = (\bar{\Theta} + e^{\zeta_t} - 1) M_t, \qquad \Theta_t = [M_{t+1} - M_t]/M_t, \tag{4}$$

where Θ_t is the growth rate of money and $\bar{\Theta}$ is the stationary gross growth rate of money.

The equilibrium intertemporal Euler condition in this model with leisure is standard; given the inflation rate π_{t+1} defined by P_{t+1}/P_t, this condition is

$$\frac{1}{R_t} = \beta E_t \left\{ \frac{c_{t+1}^{-\sigma} x_{t+1}^{\psi(1-\sigma)}}{c_t^{-\sigma} x_t^{\psi(1-\sigma)}} \frac{1}{\pi_{t+1}} \right\}. \tag{5}$$

A log-linearized form of this equation, with over-bars indicating net rates, and $\overline{g}_c$ and $\overline{g}_x$ indicating the net growth rate of the subscripted variables, is then

$$\overline{R}_t - \overline{R} = E_t \left(\overline{\pi}_{t+1} - \overline{\pi} \right) + \sigma E_t \left(\overline{g}_{c,t+1} - \overline{g} \right) - \psi \left(1 - \sigma \right) E_t \overline{g}_{x,t+1}. \tag{6}$$

The Euler condition looks similar to some form of a Taylor (1993) equation in which the growth in consumption and in the leisure rate replace the so-called "output gap" or real interest rate components of the model. However, the coefficient on the inflation term is Fisher-like, at one, rather than Taylor-like at above one.

Money has not been introduced into the Euler equation but now it will be in an alternative equilibrium condition of the model that can be the focus of interest rate determination in the estimation model. Using the cash-in-advance constraint, it is true that

$$E_t \left(\frac{M_{t+1}}{M_t} \right) = E_t \left(\frac{P_{t+1} c_{t+1}}{P_t c_t} \right).$$

Using the money supply growth notation of Θ_t, we can substitute this into the Euler equation and so eliminate consumption such that the resulting log-linearized equation is given by

$$\overline{R}_t - \overline{R} = \left(1 - \sigma \right) E_t \left(\overline{\pi}_{t+1} - \overline{\pi} \right) + \sigma E_t \left(\Theta_{t+1} - \bar{\Theta} \right) - \psi \left(1 - \sigma \right) E_t \overline{g}_{x,t+1}.$$

This model brings in the money supply growth rate into the interest rate determination, but the coefficient on the inflation term will either be less than one if the CES utility coefficient is $\sigma < 1$, or it will be negative if $\sigma > 1$. This aspect makes the model inconsistent with results related to Taylor rule estimations, and those found in our own empirical analysis below.

2.2 Endogenous Velocity Extension of the CIA Economy

Extended to include an exchange credit substitute for cash, as in Benk et al. (2010), we use a decentralized "banking time" approach that uses the financial intermediation service sector to produce the exchange credit using a CRS technology.[2] This financial

[2] This has been employed steadily in the banking literature since Clark (1984), Hancock (1985) and and and Humphry and Berger(1997).

sector produces real exchange credit q_t to purchase goods during the period. The consumer buys this credit service and pays off the exchange debt at the end of the period, out of the consumer's deposits d_t still remaining in the bank after cash withdrawals at the beginning of the period. The consumer's exchange constraint is $q_t + \frac{M_t+T_t}{P_t} = d_t$. The deposit constraint is that $c_t = d_t$ as all deposits are spent each period (the consumer self-produces without intermediation intertemporal savings and investment through the accumulation of physical capital k_t and its rental to the goods producer). Defining $m_t \equiv \frac{M_t+T_t}{P_t}$, and $a_t \equiv \frac{m_t}{c_t} = 1 - \frac{q_t}{c_t}$, the cash purchases are $a_t c_t$ and credit purchases are $(1-a_t)\,c_t$.

Because there is time spent working in the bank sector, l_{Qt}, the allocation of time constraint is now $l_{Gt} + l_{Ht} + l_{Qt} + x_t = 1$. The bank sector production function for credit is $q_t = A_Q e^{v_t} (l_{Qt} h_t)^{\gamma}\, d_t^{1-\gamma}$, where d_t are deposits made by the consumer in the bank at the beginning of each period, and with $A_Q \in R_+$, $\gamma \in [0,1)$; and v_t is a shock to banking with a similar specification as in equation (1). This shocks extends the shock vector to $S_t' = [z_t\ \zeta_t\ \nu_t]'$, with added parameters of autocorrelation φ_ν and with $\varepsilon_{St}' = [\epsilon_{zt}\ \epsilon_{\zeta t}\ \epsilon_{\nu t}]' \sim N(\mathbf{0}, \mathbf{\Sigma}')$, with $\mathbf{\Sigma}'$ the revised covariance matrix and with correlation between shocks possible.[3]

The goods producer problem is the same as in the last section with cash-only. Using this extended framework, Davies et al. (2013) show that the resulting intertemporal capital Euler condition is extended relative to equation (5) to be

$$1 = \beta E_t \left\{ \frac{c_{t+1}^{-\sigma} x_{t+1}^{\psi(1-\sigma)}}{c_t^{-\sigma} x_t^{\psi(1-\sigma)}} \frac{\tilde{R}_t}{\tilde{R}_{t+1}} \frac{R_{t+1}}{\pi_{t+1}} \right\}, \tag{7}$$

where $\tilde{R}_t$ represents one plus a 'weighted average cost of exchange' as follows:

$$\tilde{R}_t \equiv 1 + a_t \bar{R}_t + \gamma (1-a_t) \bar{R}_t.$$

Since γ is the coefficient of labor in the production of credit q_t, and it is less than one, the average cost of exchange is lowered by using credit, even as scarce time is used up in the process of avoiding the inflation tax (which is not socially optimal, but is privately optimal for the consumer). In contrast, with a simple CIA constraint, $a_t = 1$, $\tilde{R}_t = R_t$, and there is not an expected future interest rate in the equation , via R_{t+1}, since $\tilde{R}_{t+1} = R_{t+1}$ in the simple CIA model and this term cancels out. This means the velocity extension of the model through providing exchange credit through the banking sector itself is responsible for bringing in the expected future interest rate into the Euler equation, as well as bringing in the expected future change in velocity itself.[4]

[3] It can be shown that this decentralized bank sector version of the model in Benk et al. (2010) gives the same equilibrium as the model with the consumer acting in part as a bank (Hicks, 1935), without that sector decentralized as in Benk et al. (2008).

[4] See Bansal and Coleman (1996) for a related model and Euler equation.

2.3 Log Linearized Interest Rate Condition with Money Supply

Log-linearization of equation (7) implies that

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\sigma E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) - \Omega\psi\left(1-\sigma\right) E_t \overline{g}_{x,t+1} \\ &\quad + \left(\Omega - 1\right)\overline{R}\frac{a}{1-a} E_t \overline{g}_{a,t+1} - \left(\Omega - 1\right) E_t\left(\overline{R}_{t+1} - \overline{R}\right),\end{aligned} \tag{8}$$

where $\Omega \equiv 1 + \frac{(1-\gamma)(1-a)}{\left(1+\overline{R}\right)\left[\gamma + a(1-\gamma)\right]} \geq 1$, and a is the BGP solution for normalized money demand: $a_t \equiv \frac{m}{c} = 1 - A_Q\left(\frac{\overline{R}\gamma A_Q}{w}\right)^{\frac{\gamma}{1-\gamma}} \leq 1$, and $\bar{R}$ the BGP solution for $\overline{R}_t$. Since $\Omega \geq 1$ (=1 only if $R = 0$ at the Friedman, 1969, optimum), the forward-looking interest rate term enters the equation, along with a velocity growth term $\overline{g}_{a,t+1}$. These extra terms drop out for $a = 1$, at $R = 0$, as the equation reduces back to the form found in the simple CIA economy in which only cash is used ($a = 1$). One clear advantage of this extension for substantiating the model through empirical work is that the coefficient on the inflation term Ω is above one as is found also in the Taylor literature. Davies et al. (2013) point out that estimation of this equilibrium condition can be observationally equivalent to estimation of a differently motivated "reaction-function" Taylor equation.[5]

A way to re-write the Euler equation with money supply in this case is again to combine it with the CIA constraint. This results in a modified log-linearized equilibrium condition of

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\sigma E_t\left(\Theta_{t+1} - \bar{\Theta}\right) - \Omega\psi\left(1-\sigma\right) E_t \overline{g}_{x,t+1} \\ &\quad + \left[\left(\Omega - 1\right)\overline{R}\left(\frac{a}{1-a}\right) - \Omega\sigma\right] E_t \overline{g}_{a,t+1} - \left(\Omega - 1\right) E_t\left(\overline{R}_{t+1} - \overline{R}\right).\end{aligned} \tag{9}$$

To interpret the above expression, now consider that on the perfect foresight BGP stationary equilibrium, with a balanced growth rate of g, it results that the nominal interest rate is directly related to the money supply growth rate:

$$\begin{aligned} 1 &= \beta\left(1+g\right)^{-\sigma}\left(1+r\right), \\ \left(1+\bar{R}\right) &= \left(1+\overline{\pi}\right)\left(1+r\right), \\ \left(1+\bar{\Theta}\right) &= \left(1+\overline{\pi}\right)\left(1+g\right), \end{aligned}$$

and so $\left(1+\bar{\Theta}\right) = \left(1+\bar{R}\right)\beta\left(1+g\right)^{1-\sigma}$, or $\bar{R} \simeq \bar{\Theta} + \rho + \left(\sigma - 1\right)g$, where $\beta \equiv \frac{1}{1+\rho}$. This BGP expression for R implies that the expected nominal interest rate follows the expected money supply growth rate plus a smaller magnitude term involving the expected growth rate.

If the forward-looking expected interest rate term is replaced simply by the expected money supply growth rate, and if the velocity term is dropped, since in the estimation below this variable is tested to be stationary (and not significant in the regimes dynamics), then we could approximate the log-linearized equilibrium condition as

$$\overline{R}_t - \overline{R} \simeq \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \left[\Omega\sigma - \left(\Omega - 1\right)\right] E_t\left(\Theta_{t+1} - \bar{\Theta}\right) - \Omega\psi\left(1-\sigma\right) E_t \overline{g}_{x,t+1}. \tag{10}$$

[5] See Alvarez et al. (2001) for a related approach within a segmented market economy with exogenous velocity.

The expectations of the variables $\overline{\pi}_{t+1}$, Θ_{t+1}, $\overline{g}_{x,t+1}$ are approximated using "naive" expectations approach by using the current time t values of the variables. This then gives an equation at the same time t for $\bar{R}_t$, $\overline{\pi}_t$, Θ_t, and $\overline{g}_{x,t}$:

$$\overline{R}_t - \overline{R} = \Omega\left(\overline{\pi}_t - \overline{\pi}\right) + \left[\Omega\sigma - (\Omega - 1)\right]\left(\Theta_t - \bar{\Theta}\right) - \Omega\psi\left(1-\sigma\right)\overline{g}_{x,t}. \tag{11}$$

The above equation implies restrictions on the parameters of the model. The four variables are R, π, Θ, and the growth in the leisure x. Given the interpretation of increases in leisure in our model as increases in "unproductively used time", we will proxy the growth in leisure by the unemployment rate, which we denote by u_t at time t.[6] This specification results in a reduction of the number of variables to the set $y_t \equiv \left[\overline{R}_t, \overline{\pi}_t, \Theta_t, u_t\right]$. The restrictions for u_t from the coefficient $-\Omega\psi\left(1-\sigma\right)$ imply that $\sigma < 1$ would give a negative coefficient for the unemployment rate given a positive leisure preference ψ and inflation parameter Ω. This latter inflation rate parameter is restricted in theory to be $\Omega \geq 1$. The money supply growth rate term coefficient of $\Omega\sigma - (\Omega - 1)$ would be negative for example if $\sigma = 0.5$, as is within the range seen in the literature, and if $\Omega = 4$, as is plausible.[7] In this case, $\Omega\sigma - (\Omega - 1) = -1$, so that the money supply growth would have a negative effect with a coefficient of -1 as found in the evidence presented below.

An important note is that a fixed point solution to the model of Section 2.2, around which a log-linearization can be done, requires that if the money supply growth rate follows a unit root then it must do so within some bounded range in order for a BGP solution to exist. As we do find such a unit root below in the data, the importance of the idea of boundedness in the money supply process comes through, thereby using this crucial concept that has been popularized by the work of Leeper and Zha (2003) and the subsequent related work.

2.4 Extended Money Supply Specification

Without loss of generality, we will extend the stochastic specification of the money supply process from a unit root process in the stock of money, M_t, to one with possible integration of order 2 in the stock of money, and order 1 in the growth rate of the money supply, Θ_t. Further we let the shock X_t be thought of as being dependent on the time t state of the three previously specified shocks in the economy as given by the vector S'_t. Then the money supply growth rate process can be postulated in a state-dependent

[6] The first order conditions of the full economy, as given in full in Benk et al. (2010), imply that $\beta E_t\left(1-x_{t+1}\right)w_{t+1}\lambda_{t+1} = -\left[\beta E_t\psi_{t+1}\left(1-\delta_H\right)-\psi_t\right]$, where λ_t is shadow price of real income, ψ_t is shadow price of human capital. This implies that the expected employment rate determines the negative of the expected growth in the value of the human capital. Our proxy is to use the unemployment rate instead of the employment rate, and so relate it to the (positive) growth in the value of the human capital, and then second to proxy the the expected growth in leisure by the growth in the value of the human capital. Therefore we proxy the growth in leisure by the unemployment rate.

[7] For example if $R = 0.06$, $\gamma = 0.1$, $a = 0.155$, then $\Omega = 4.0$.

fashion as

$$\Theta_{t+1}\left(S'_{t+1}\right) = \Theta_t\left(S'_t\right) + X_t\left(S'_t\right), \tag{12}$$

where $X_t\left(S'_t\right)$ is i.i.d. and a function of the composite shocks coming from the goods and bank sector productivity and the money shock. This implies that the central bank sets the money supply growth rate in a state-dependent way through the $X_t\left(S'_t\right)$ term. We assume a boundedness in this process, and note that by using data series on endogenous variables it is possible to back out the implied money supply process.[8]

3 The Empirical Model

The analysis presents a system framework with the possible presence of regimes, in a Markov-Switching Cointegrated Vector Error Correction Model (MS-VECM) for the US data period of 1960-2012.

3.1 Data

The data is monthly for the United States from 1960.1 to 2012.12, as given in the Federal Reserve Bank of St. Louis FRED database. This comprises the Federal funds rates for $\bar{R}_t$; the percentage change in the CPI for the inflation rate $\bar{\pi}_t$ [$\bar{\pi}_t = (\Delta_{12} cpi_t)100$) where cpi_t is the log transformed consumer price index ($\ln CPI$)]; the log of the unemployment rate u_t; the growth rate in $M2$ [($\Theta = \Delta_{12} \ln M2)100$] for the monetary aggregate. Figure 1 graphs each of the four variables.

Each series is well characterized as an I(1) process (see Appendix A). We check for the presence of a unit root by means of the ADF test and the DF-GLS test (Elliott et al., 1996), allowing for an intercept as the deterministic component. The unit root null cannot be rejected at the 5% level in all cases. KPSS stationarity tests (Kwiatkowski et al., 1992) confirm this result. Differencing the series induces stationarity. These results are confirmed in the multivariate framework.

3.2 A VECM representation of the data set

Therefore we assume that the true dynamics can be approximated by a VAR(k) system, which can be more conveniently written as a VECM($k-1$):

$$\Delta y_t = \upsilon + \Pi y_{t-1} + \sum_{j=1}^{k-1} \Upsilon_j \Delta y_{t-j} + \Sigma \varepsilon_t \ , \tag{13}$$

where $y_t = \begin{bmatrix} \overline{R}_t & \overline{\pi}_t & \Theta_t & u_t \end{bmatrix}'$, υ is the vector of intercept terms, Υ_j are matrices containing short-run information, while Π is a vector with the long-run information

[8] Benk et al. (2005, 2010) identify such goods and bank sector productivity shocks, along with the money supply shock; other bank shocks have been postulated such as Jermann and Quadrini (2012). We do not back out the state-dependent money supply process in this paper, but instead offer it in terms of the cointegrating vector evidence along with the three MS regimes.

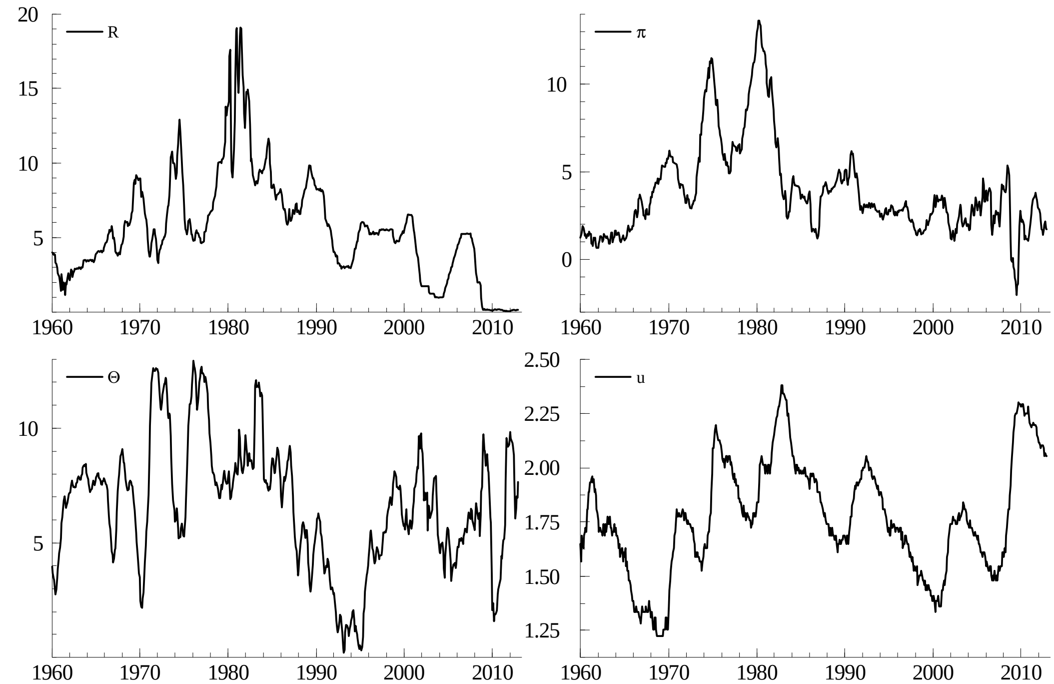


Figure 1: Federal Fund Rate, inflation rate, rate of growth of M2, log of unemployment rate.

of the data and $\Sigma\varepsilon_t$ is a vector of errors with $\varepsilon_t \sim i.i.d.N(0, I)$. The assumption is that the reduced-form shocks follow a multivariate normal distribution, $\Sigma\varepsilon_t \sim N(0, \Phi)$, where Φ denotes the variance-covariance matrix of the errors.

After testing to determine the maximum lag length of the system (13), we apply Johansen's (1988, 1991) approach by estimating a VAR(6) and testing for the reduced rank of Π.(see Appendix B).[9] Here we define $\Pi \equiv \alpha\beta'$, where α and β are (4×1) vectors. The α is a vector of "loading coefficients" describing each variable's speed of adjustment back to the long run equilibrium if it is significant, and lack of adjustment if insignificant. The β vector contains the cointegrating coefficients of each variable. Table 1 reports the results of the cointegration test (i.e. the trace and the maximum eigenvalue test). It indicates that the long-run matrix Π has a reduced rank $(r = 1)$. Hence, we conclude that there is exactly one cointegrating relationship amongst the four variables. Table 2 reports the α and β coefficients for each variable of the cointegrating vector.

[9] This finding is corroborated by looking at the roots of the companion matrix of the chosen VAR(6), which show that there are three common trends.

Table 1: Test for cointegrating rank

Rank	0	1	2	3
Trace test [Prob]	79.50[0.000]**	32.35[0.098]	16.12[0.172]	5.20[0.272]
Max test [Prob]	47.15[0.000]**	16.23[0.293]	10.92[0.267]	5.20[0.272]
Trace(T-nm) [Prob]	76.46[0.000]**	31.11[0.129]	15.50[0.203]	5.00[0.294]
Max(T-nm) [Prob]	45.35[0.000]**	15.61[0.339]	10.51[0.301]	5.00[0.293]

Note. The trace test and the max test are the log-likelihood ratio tests (LR), which are based on the four eigenvalues (0.072, 0.025, 0.017 and 0.008). The VAR tested for cointegration is a VAR(6) with an intercept in the cointegrating vector. The row denoted as rank reports the number of cointegrating vectors, and [prob] indicates the p-value computed from critical values by Doornik (1998). The last two rows report small sample correction.

Table 2: Cointegrated coefficients and loading coefficients

	Cointegrating coefficients β'	Loading coefficients α
$\bar{R}_t$	$\mathbf{1}$	$\boldsymbol{\alpha}_R = -\mathbf{0.012}\ (\mathbf{0.004})$
$\bar{\pi}_t$	$-\mathbf{2.519}\ (\mathbf{0.295})$	$\boldsymbol{\alpha}_\pi = \mathbf{0.002}\ (\mathbf{0.003})$
Θ_t	$\mathbf{0.927}\ (\mathbf{0.282})$	$\boldsymbol{\alpha}_\Theta = -\mathbf{0.011}\ (\mathbf{0.003})$
u_t	$\mathbf{10.952}\ (\mathbf{2.913})$	$\boldsymbol{\alpha}_u = -\mathbf{0.001}\ (\mathbf{0.0002})$
$Const.$	$-\mathbf{21.475}\ (\mathbf{5.212})$	

Note. The standard errors are presented in the round parentheses

Table 3 presents results of the cointegrating vector with two additional restrictions. We test if Θ is a relevant variable for cointegration, and the LR test on $\beta_\Theta = 0$ strongly rejects the hypothesis that it is not relevant: $\chi^2(1) = 7.301[0.0069]^{**}$. In addition, as the coefficient α_π is not significantly different from zero, we also test the restriction $\alpha_\pi = 0$, which is not rejected implying that π is weakly exogenous. Further, testing the hypothesis that $\beta_\Theta = 1$ results in it not being rejected ($\chi^2(2) = 0.44821[0.7992]$). Tables 2 and 3 both show that with reference to the entire period all the variables react to the equilibrium error with the expected sign, expect the inflation rate $\bar{\pi}_t$ which is weakly exogenous.[10]

[10] This last result appears consistent with setting the level of the *BGP* inflation rate (the de facto target) largely by the exogenous *BGP* mean rate of money supply growth within the cash-in-advance economy.

Table 3: Multivariate cointegration analysis

	Cointegrated coefficients β	Loading coefficients$a$
$\bar{R}$	$\mathbf{1}$	$\boldsymbol{\alpha}_R = \mathbf{-0.011}\ (\mathbf{0.0038})$
$\bar{\pi}_t$	$\mathbf{-2.572}\,(\mathbf{0.304})$	$\boldsymbol{\alpha}_\pi = \mathbf{0}$
Θ_t	$\mathbf{1}$	$\boldsymbol{\alpha}_\Theta = \mathbf{-0.011}\ (\mathbf{0.0029})$
u_t	$\mathbf{12.145}\,(\mathbf{3.074})$	$\boldsymbol{\alpha}_u = \mathbf{-0.001}\ (\mathbf{0.0002})$
$Const.$	$\mathbf{-23.900}\,(\mathbf{5.395})$	
	Test of weak exogeneity	LR test of restrictions:
Restriction:	$\alpha_R = 0$	$\chi^2(1) = 6.2675[0.0123]^*$
Restriction:	$\alpha_\pi = 0$	$\chi^2(1) = 0.4375[0.5083]$
Restriction:	$\alpha_\Theta = 0$	$\chi^2(1) = 9.4585[0.0021]^{**}$
Restriction:	$\alpha_u = 0$	$\chi^2(1) = 11.336[0.0008]^{**}$

Note.The standard errors are presented in the round parentheses, while the p-values are reported in the square brackets

The results imply for the restricted cointegrating vector, with the nominal interest rate put on the lefthandside, a coefficient of -1 for the money supply growth rate on the nominal interest rate, as in our example calibration of the extended cash-in-advance economy. In addition the inflation rate has a significant positive effect with a coefficient of the magnitude seen in many Taylor type estimations, and unemployment a negative effect as in some Taylor type estimations. The results show that the estimation is consistent with the theory in Section 2 in which the money supply growth rate plays a key role. The results suggest possible misspecification bias for related estimation that omits the money supply.

3.3 Three State Markov-switching VECM Analysis

Starting with the estimated long-run cointegrating vector, we extend analysis by including potential regime shifts in short-run dynamics. We find different nonlinearities in the responses of $\bar{R}_t$, $\bar{\pi}_t$, Θ_t and u_t to the equilibrium error under different regimes.[11] We follow Krolzig (1997, 1998) who employs a Markov regime-switching vector error correction model (MS-VECM) to allow for state dependence in the parameters. Krolzig's procedure consist of a two-step approach: the first step corresponds to a cointegration analysis in a standard linear model while in the second step the analysis applies the Markov-switching methodology to account for regime shifts in the short-run parameters of the estimated VECM. This gives a multivariate linear system of non-stationary

[11] See also the regime switching Taylor rules of Valente, 2003a; Francis and Owyang, 2005; Assenmacher-Wesche, 2006; Castelnuovo et al., 2012; and Trecroci and Vassalli, 2010. Smets and Wouters (2007) propose an example of a monetary policy reaction function that is characterised by the presence of a non-stationary process in the "usually constant" term. Also see Woodford (2008) for a model with non-stationary targets and some comments on this feature of the Smets and Wouters's (2003) model.

time series that is subject to regime shift.[12]

The Markov regime-switching model is based on the idea that the parameters of a VAR depend upon a stochastic, unobservable regime variable $s_t \in (1, ..., N)$. Therefore, it is possible to describe the behavior of a variable (or the behavior of a combination of variables) with a model that describes the stochastic process that determines the switch from one regime to another by means of an ergodic Markov chain defined by the following transition probabilities:

$$p_{ij} = \Pr(s_{t+j} = j \,| s_t = i), \quad \sum_{j=1}^{N} p_{ij} = 1, \quad i, j \in \{1, ..., N\}$$

The cointegrating relations are included in the MS(N)-VECM($k-1$) as exogenous variables, which are assumed to remain constant, where k denotes the number of lags and N the number of regimes.[13] There are many types of MS-VAR models and in this framework the model selection is more complex than in a linear model. We have to decide the maximum lag, which parameters are allowed to vary and how many regimes are to be estimated. The letters following MS stand for the respective parameters varying, specifically: I for the intercept, A for the short-run coefficients, and H for the covariance matrix. The Markov-switching MSIAH-VECM that generalizes the system (13) to account for regime shifts in all these components has the following specification:

$$\Delta y_t = \upsilon(s_t) + \alpha(s_t)\beta' y_{t-1} + \sum_{j=1}^{k-1} \Upsilon_j(s_t)\Delta y_{t-j} + \Sigma(s_t)\varepsilon_t, \qquad (t = 1, .., T) \tag{14}$$

where[14] $\Sigma(s_t)\varepsilon_t \sim N(0, \Phi(s_t))$, $\Phi(s_t) = \Sigma(s_t)\Sigma'(s_t)$, $s = 1, .., z$ and the parameters $\upsilon(s_t)$, $\alpha(s_t)$, $\Upsilon_j(s_t)$, and $\Phi(s_t)$ describe the dependence on a finite number of regimes s_t. Hansen and Johansen (1998) have shown that shifts in $\upsilon(s_t)$ are decomposed into shifts in the mean of the equilibrium error and shifts in the short-run drifts of the system.

First we proceed to investigate the presence of nonlinearities allowing regime shifts in the unrestricted intercept (I), in the adjustment coefficients (A), and in the variance-covariance matrix (H), MSIAH-VECM (also known as MSIAH-VARX, where X means that in specification (14) the equilibrium relation obtained in the first step ($\beta' y_{t-1}$) is exogenous). The model captures shifts in the mean of the equilibrium error along with

[12] The MS-VAR model by Krolzig (1997) is a multivariate generalisation of Hamilton (1989) to non-stationary cointegrated VAR systems. For this analysis it can be assumed that the error term is not normally distributed; Johansen (1991, p. 1566) shows that the assumption of Gaussian distribution is not relevant for the results of the asymptotic analysis. Saikkonen (1992) and Saikkonen and Luukkonen (1997) show that most of the asymptotic results of Johansen (1988 and 1991) for estimated cointegration relations remain valid and can be extended to include the data generated by an infinite non-Gaussian VAR.

[13] In this contest the usual estimation method of parameters is the maximum likelihood and, since the state variable s_t is unobservable, Hamilton (1989) suggests using a maximum likelihood estimation technique via an Expectation Maximization (EM) algorithm. For a detailed description see Krolzig (1998).

[14] Model (14) is indicated as MSIAH(M)-VECM($k-1$) and could be considered the more general model in terms of changing coefficients.

shifts in the drift and in the variance-covariance matrix of the innovations. At the same time we relax the assumption of linear adjustment towards the equilibrium, letting the vector of adjustment coefficients $\alpha(s_t)$ and the matrices of the autoregressive part also be regime-dependent.

We choose the number of regimes and the model in relation to the possible combination of changing parameters, amongst the MSIAH, MSAH, MSIH and MSH alternatives. Model selection is related to Krolzig (1997), Sarno and Valente (2000), Valente (2003b). As a first step, within a given regime (M) and a given MS specification, we choose the best model in terms of maximum lag using the Information Criteria (IC). We then compare the various MS specifications[15], choosing the model that dominates in terms of the IC and LR (log-likelihood ratio) tests. The model selection procedure is repeated for different regimes and, finally, the chosen models with different regimes are compared and selected with the usual IC.[16]

Appendix C shows that comparing the IC reported in Table 1C and in Table 2C it is difficult to choose between the models MSAH(3)-VECM(1) and MSIAH(2)-VECM(1) and so based on the LR test we choose the more general MSIAH(3)-VECM(1). Note that there is no relevant difference in terms of the dating of the regimes, and also there is no difference with reference to all other important information related to the concept of weak exogeneity and volatility. Appendix D presents further results for the less statistically preferred two-state Markov model which leaves out a key aspect of our results: the Regime 3 that we report in this section.

We report the results of the three-state Markov-switching VECM of the MSIAH(3)-VECM(1) form. All the tests support the non-linearity (LR linearity test: 1327.2753, $\chi^2(68) = [0.0000]^{**}$, $\chi^2(74) = [0.0000]^{**}$). Moreover, the Davies (1987) upper bound test does not reject the non-linear model: $DAVIES = [0.0000]^{**}$.

[15] This procedure was done for each combination of changing parameters (MSIAH, MSAH, MSIH, MSH). Results are reported in Appendix C.

[16] It is important to note that formal testing is difficult here because of an identification problem. See on this point Krolzig (1997), Sarno et al. (2004). For extensive discussions of the problems related to LR testing in this context, see Hansen (1992, 1996) and Garcia (1998).

Table 4: Estimated coefficients in the non linear VECM(1)

Regime 1	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.v$	**0.758**	0.150	0.021	**0.041**
$\Delta\overline{R}_{t-1}$	**0.319**	0.059	$-$**0.119**	-0.001
$\Delta\overline{\pi}_{t-1}$	0.033	0.179	-0.194	-0.004
$\Delta\Theta_{t-1}$	**0.741**	$-$**0.269**	**0.237**	0.001
Δu_{t-1}	$-$**12.01**	-1.302	-0.471	0.161
$\beta' y_{t-1}$	$-$**0.029**	-0.008	-0.001	$-$**0.001**
SE (Reg.1)	1.037	0.397	0.407	0.032
Regime 2	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	0.001	-0.112	**0.221**	**0.026**
$\Delta\overline{R}_{t-1}$	**0.481**	0.106	$-$**0.219**	$-$**0.016**
$\Delta\overline{\pi}_{t-1}$	**0.103**	**0.314**	$-$**0.190**	0.004
$\Delta\Theta_{t-1}$	0.042	-0.021	0.575	0.004
Δu_{t-1}	$-$**1.663**	0.032	$-$**0.272**	$-$**0.217**
$\beta' y_{t-1}$	-0.0002	0.005	$-$**0.009**	$-$**0.001**
SE (Reg.2)	0.205	0.252	0.252	0.026
Regime 3	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	**0.094**	$-$**0.656**	**0.729**	-0.015
$\Delta\overline{R}_{t-1}$	**0.660**	**0.946**	$-$**0.865**	-0.025
$\Delta\overline{\pi}_{t-1}$	-0.013	**0.345**	$-$**0.315**	-0.006
$\Delta\Theta_{t-1}$	0.009	0.005	**0.308**	0.006
Δu_{t-1}	0.127	-1.621	-1.140	0.195
$\beta' y_{t-1}$	$-$**0.003**	**0.026**	$-$**0.029**	0.001
SE (Reg.3)	0.051	0.458	0.625	0.021

Note. Bold characters mean rejection of the null hypothesis of zero coefficients at the 95% confidence level or higher.

Table 4 presents the distinct set of the estimated parameters of the VECM in each regime, endogenously separated by Markov-switching methodology. The three distinct regimes provide a picture that differs with respect to the coefficients of adjustment to the equilibrium error, to the variance-covariance matrix of the innovations and to the cyclical phase. Regime 1, in Figure 3, exhibits a higher interest rate volatility (SE= 1.037, see Table 4) and is strongly characterized by the adjustment of the interest rate to the equilibrium error (with a coefficient of -0.029, see Table 4) and by the absence of an adjustment of money growth (the coefficient -0.001 is not significant, see Table 4), which is weakly exogenous as the inflation rate. In general, we can observe that the dating of regime 1 probabilities is consistent with the findings of Sims and Zha (2002),[17] Francis and Owyang (2005) and also with models for the dating of recession periods according to NBER (see Figure 3).[18] Regime 1 captures roughly all of the post 1960 recessions except 1991, and adds one extraneous short period around 1985. [19]

The second regime is characterized by moderate volatility of all of the variables (see the SE values for regime 2 in Table 4) and tends to coincide with NBER expansions (see Figure 4). The interest rate and inflation rate do not adjust to the equilibrium error thereby becoming weakly exogenous (the coefficients -0.0002 and 0.005 are not significant, see Table 4), while money growth and the unemployment rate adjust to the equilibrium error (with a coefficient of, respectively, -0.001 and -0.009, see Table 4). Moreover, the inflation rate is now a strongly exogenous variable (since it only adjusts to $\Delta\overline{\pi}_{t-1}$ with a coefficient of 0.314).

Regime 3, as shown in Figure 5, prevalently captures the more recent periods, from 2004 to 2012. Both money growth and the interest rate adjust to the equilibrium error (with a coefficient of, respectively, -0.029 and -0.003, see Table 4); the coefficient of adjustment of money growth is higher than in regime 2 ($0.029 > 0.009$, see Table 4), while the interest rate adjustment is lower than in regime 1 ($0.003 < 0.029$, see Table 4). This is also the only regime where the inflation rate is not weakly exogenous (with a coefficient of adjustment to the equilibrium error of 0.026). The unemployment rate becomes a strongly exogenous variable (it shows no adjustment to all of the variables, showing a random walk behavior). This regime exhibits very low volatility in the interest rate (SE= 0.051, see Table 4) and higher volatility of money growth (SE= 0.625, see Table 4) and the inflation rate (SE= 0.458, see Table 4). This is a regime where a negative real interest rate coincides with its occurrence in 1971, and after 2003, although it misses the 1980 negative real interest rate by a couple of years.

Table 5 reports the estimated transition matrix and the regime properties. Roughly identifying regime 1 with NBER recessions and regime 2 with NBER expansions, as in Figures 3 and 4, Table 5 shows that: a) there is an higher probability to pass from a recession to an expansion than vice versa; b) there is an higher probability to persist in expansions than in recessions; c) the probability to pass to regime 3 when the economy is in expansion is lower than during recessions; d) when the economy is in regime 3, there is a higher probability to pass to an expansion than to a recession period.

[17] See State 3 in Figure 1, pag 6.

[18] See Hamilton (1989). Moreover, we observe that the first regime mostly coincides with the dating that we found for the first regime in the two-state Markov-switching VECM. This confirms the robustness of the identification of regime 1.

[19] This includes all of the "inflation scare" periods that were indicated by Goodfriend (1993) and Goodfriend and King (2005). Goodfriend (1993) indicates the period between 1979.12 and 1980.2 as the first inflation scare, the period between 1981.1 and 1981.10 as the second inflation scare, and the period between 1983 and 1984 as the third inflation scare, and 1987 as the fourth inflation scare.

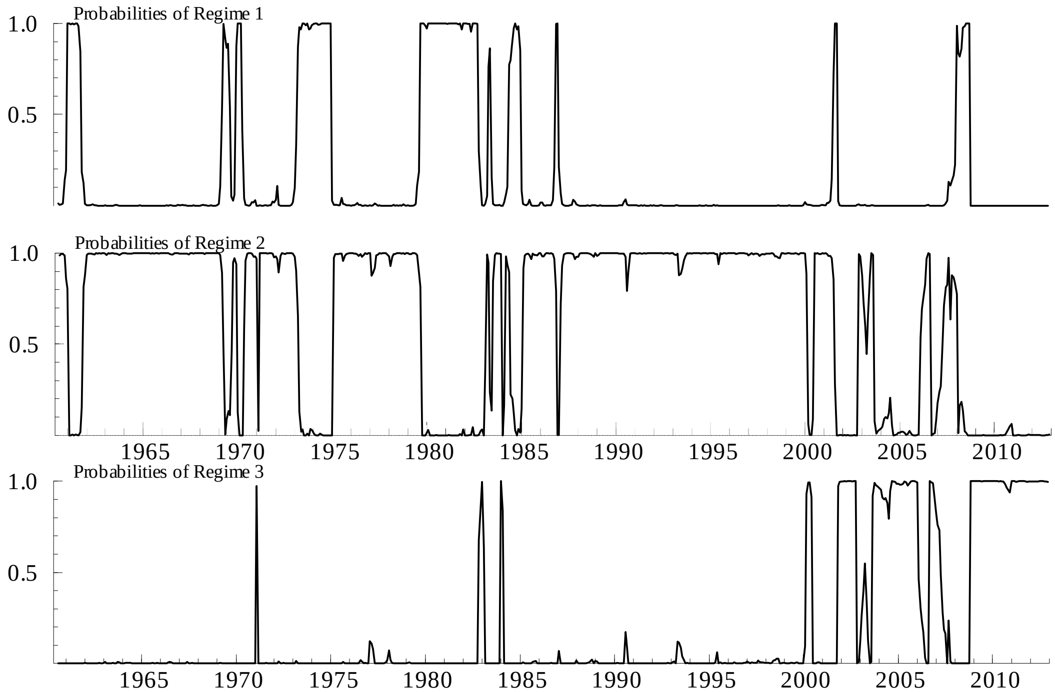


Figure 2: Conditional (smoothed) probabilities of the three regimes obtained from MSIAH(3)-VECM(1) for $\Delta\overline{R}_t$, $\Delta\overline{\pi}_t$, $\Delta\Theta_t$, and Δu_t with the equilibrium error $\beta' y_t = \overline{R}_t - 2.6\overline{\pi}_t + \Theta_t + 12.2u_t$ restricted as an exogenous variable.

Table 5: Transition probabilities and Regime properties

Transition probabilities	p_{1i}	p_{2i}	p_{3i}
Regime 1	0.89	0.03	0.0002
Regime 2	0.08	0.96	0.08
Regime 3	0.03	0.02	0.92
Regime properties	*nObs*	*Prob*	*Duration*
Regime 1	103.6	0.161	9.35
Regime 2	413.4	0.648	22.94
Regime 3	112.0	0.191	11.94

3.4 Robustness: Rolling trace test

Our results for the US from 1960-2012 show how a crucial property of the nominal rate - its unit root component- is not usually treated in conjunction with the cointegration approach in estimating similar relations. Granger and Newbold (1974) and Phillips (1986) show that a static regression in levels, when some of the variables in the regression have unit roots, is spurious. Evidence of non-stationarity of the typical Taylor variables for US data has long been reported such as by Bunzel and Enders (2005)

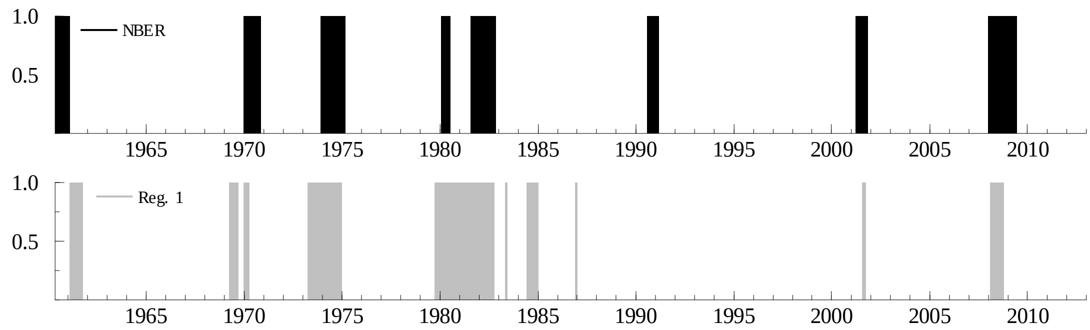


Figure 3: NBER recession dates (shadowed black areas) compared with smoothed probabilities of regime 1 (shadowed grey areas).

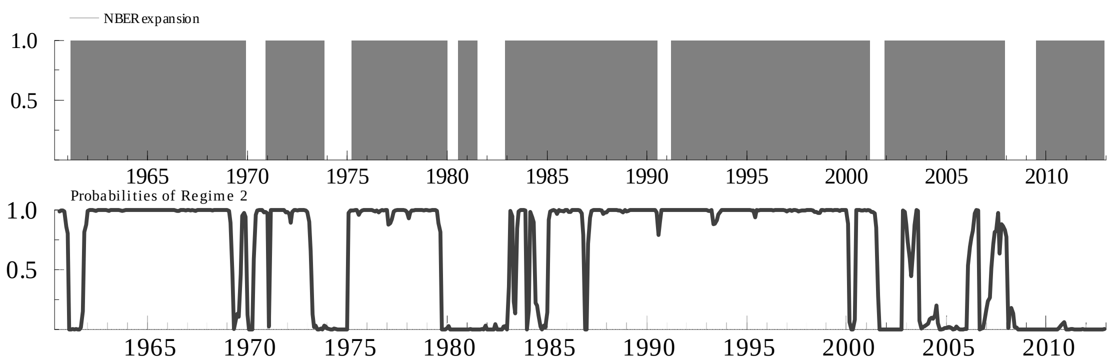


Figure 4: NBER expansions dates (shadowed grey areas) compared with smoothed probabilities of regime 2.

and Siklos and Wohar (2006). In confirmation of this issue, Gerlach-Kristen (2003) and Österholm (2005) find signs of instability, misspecification and inconsistencies in estimated Taylor rules, mainly due to mistreatment of the non-stationarity of the data.

It appears that traditional estimation without money in essence sweeps components of the cointegrating relation into the error term and/or the constant term.[20] During sub-periods with a stationary money growth process, it may be that a traditional relation performs well, while during longer periods when the money supply growth rate tests as I(1) or near I(1), the estimation with money growth would be expected to show better results. So we perform a rolling cointegration trace test with money and without money, plus the other three variables, and compare the results together with a rolling unit root test on money growth.

The rolling window technique (Rangvid and Sorensen, 2002) is based on keeping constant the size of the sub-sample and then rolling through the full sample both the first and last observation in the subsample. The size of the sub-sample is thus

[20] An interest rate relation with output and inflation as in a standard Taylor rule does not necessarily identify an interest rate reaction function (see also Orphanides, 2003) or a central bank reaction function (see Minford et al., 2002).

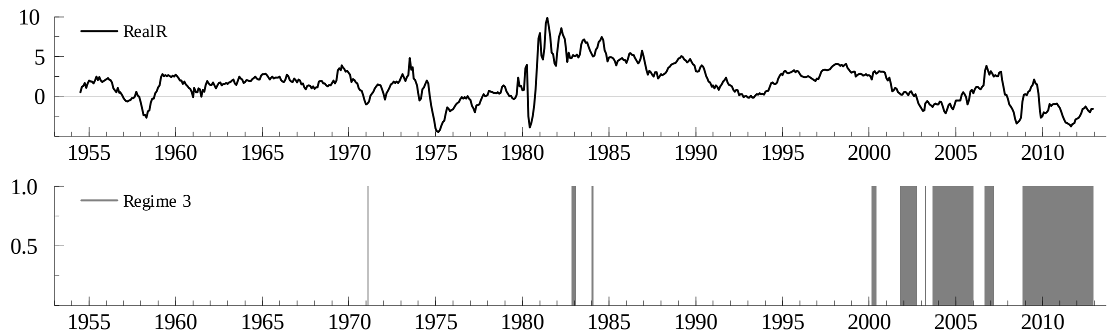


Figure 5: Regime 3 compared with the real interest rate.

a constant fraction of the size of the full. The rolling window focuses on changes in the presence of cointegration during the full sample, and provides an adept tool to investigate the presence of common stochastic trend along the period. As a type of dynamic cointegration analysis, it is a more powerful methodology with respect to recursive techniques since, as shown by Rangvid and Sorensen (2002), the expansion of the sample size in the Johansen (1991) cointegration test provides increasing values of the trace statistics. On the contrary, an increasing values for the rolling trace test could be interpreted as an increasing support for cointegration,

The continuous plot of trace test statistics for a rolling, fixed length, window provides information about the time varying pattern of the number of cointegrating vectors and the force towards convergence, expressed by the magnitude of the trace coefficient. The test statistics are calculated for a rolling 150 observations (which corresponds to 12 years and half) time window[21] by adding one observation to the end and removing the first observation and so on. That is, starting with observations 1–150, we calculate the first trace test statistics; then, we calculate the trace tests for observations 2–151, 3–152, and so on. The sequences of these statistics are scaled by their 5% critical values[22].

Figure 6 plots the scaled trace test statistics for the null hypothesis r = 0, against the alternative r = 1 (one cointegrating vector). A value of the scaled test statistic above one means that the corresponding null hypothesis can be rejected at the 5% level for the specified sub-sample period. The graph refers, respectively, to the cointegrating relation between R, π, Θ, u (the black continuous line) and between R, π, u (the dashed line). Figure 6 shows evidence of a stable cointegrating relation for both up to the end of the 1982, but a different behavior after that date. More precisely, cointegration in the formulation without money disappears after 1982 and this implies that such relations, estimated as static relations, are candidate to be spurious regressions; this is true even if a smoothing term is provided in it. On the contrary, estimation with money growth

[21] Several trials with larger windows and various lags in the VAR specification have been made with similar results.

[22] We will compute the critical values for the test using MacKinnon-Haug-Michelis (1999) p-values.

shows the presence mainly of stable cointegration, with the only multi-year exception being from 1991 to 1994.

Therefore we consider the reported results as evidence that the static equation without money estimated from the beginning of the 80's is candidate to be a spurious regression while the smoothing version is misspecified since the Engle-Granger (1987) theorem asserts that this dynamic specification is admitted only in presence of cointegration between the involved variables. On the contrary, the nominal interest rate equation with money growth suffers little from this misspecification in that cointegration dominantly appears to exist.

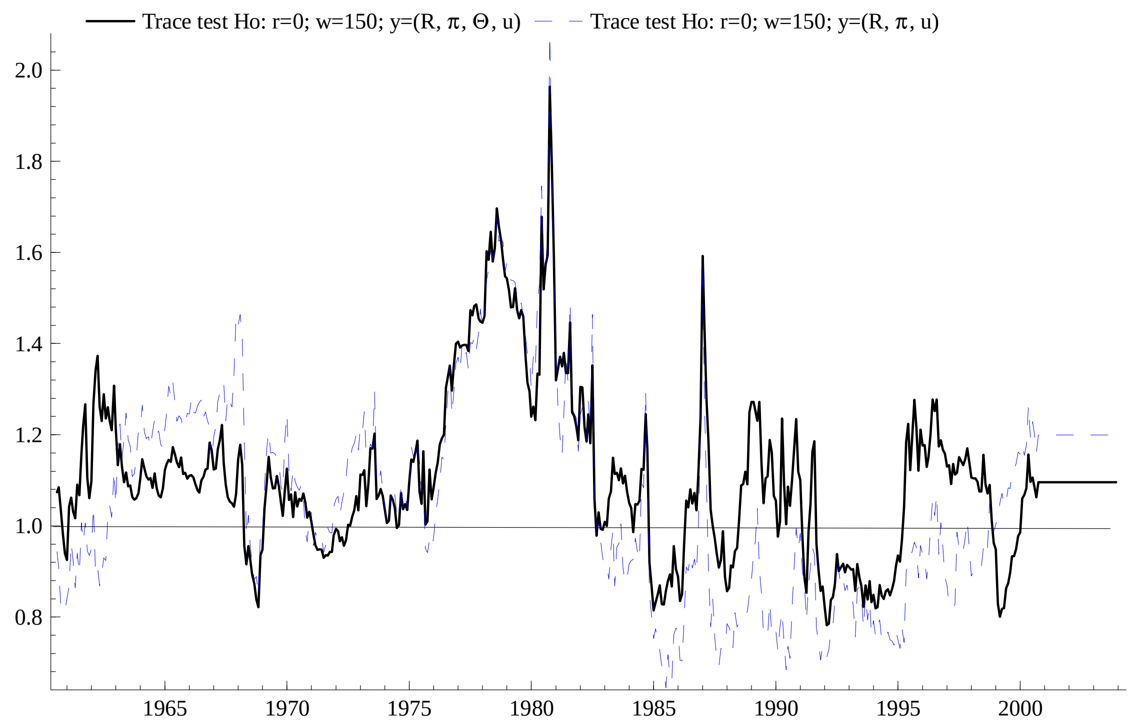


Figure 6: Rolling Trace test computed for a window equal to 150; with Euler relation, of R, π, Θ, u (the black continuous line) and without money of R, π, u (the dashed line).

Figure 7 reports the rolling unit root tests that give an insight on the dynamics of the non-stationarity for all the four variables. Here again the test was normalized and when it is above 1 non-stationarity is rejected. Therefore, we can see that all the variables are I(1) along all the period. Moreover, the overall period confirms that the nominal interest rate estimation without the money supply growth rate is misspecified over this postwar US period. [23]

[23] This is in line with Minford (2002) who notes that "Taylor (1999) himself emphasized that his rule mimicked the interest rate behavior one would expect from a k% money supply rule".

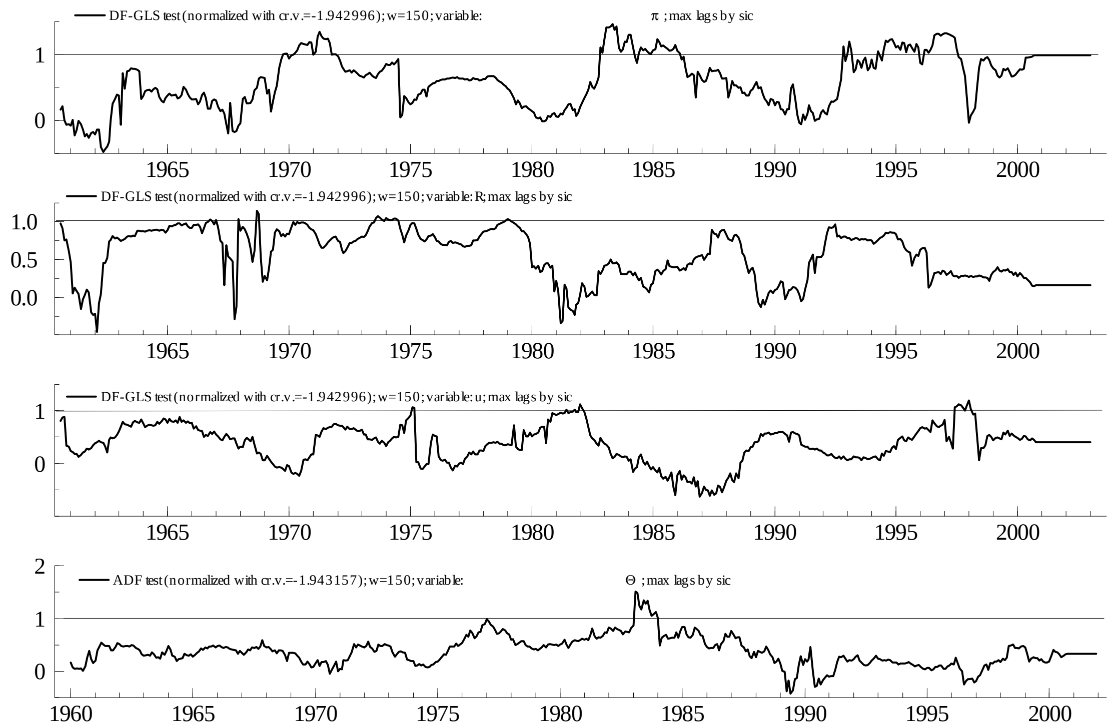


Figure 7: Rolling unit root tests computed for π, R, u and Θ for a window equal to 150.

In Figure 8 and 9 we report the rolling trace test for all possible trivariate and pairwise combinations of the four variables. The Figure 8 analysis for the trivariate case shows that there is no clear stable cointegration in all combinations, with the only exception as discussed being for the estimation without money in the first part of the sample period. Figure 9 shows that there exists no stable pairwise cointegration.

4 Discussion

Our unconventional inclusion of the money supply variable comes from a per se fiscal theory of money supply as money is simply being provided as a means of government revenue that is lump sum transferred back to the representative agent. Having clarified its approach as being independent of a strict central bank reaction function, the estimation results are consistent with following up on the suggestion by Sims and Zha (2006) that there may exist omitted variable bias in traditional estimates of the Taylor rule. We do this while accounting for the non-stationarity of the variables and getting Markov regimes consistent with the finance literature identification of contractions and expansions as regimes, in addition to our unconventional regime that is similar to the third "lost decade" MS regime of Gillman et al. (2014). But here we interpret the third regime in a monetary context by focusing on the negative ex post real interest rates that characterize it.

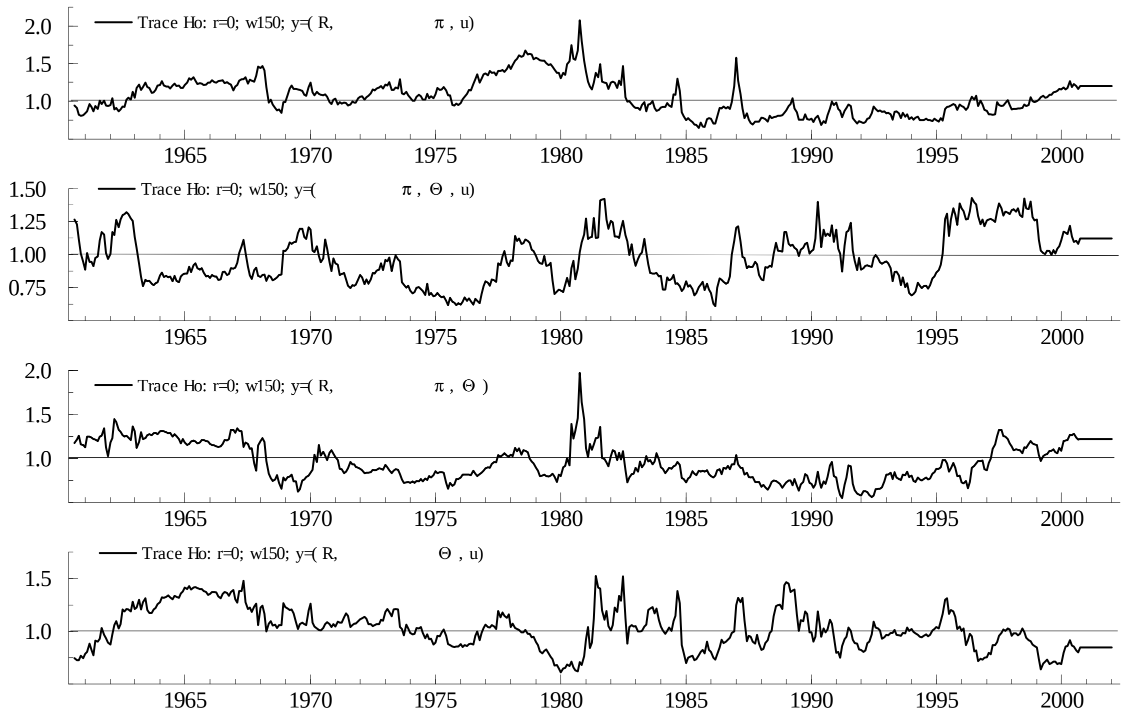


Figure 8: Rolling trace test for all possible trivariate combinations of the four variables Θ, π, R and u.

By adding the money supply growth rate, the cointegration results allow an intuitive interpretation of the economics at work in the model. Figure 10 graphs the actual money supply growth rate of M2 minus the inflation rate in the dashed line. It is clear that with real money demand defined as the money stock divided by the price level, the growth rate in the real money demand equals the growth rate in the money supply minus the inflation rate. Therefore Figure 10 is graphing the growth rate in the real money demand assuming clearing in the money market, or more simply the actual money supply growth rate minus the actual inflation rate. Comparing Figure 10 to the equilibrium error, shown as the solid line, a strong correlation results of 0.80.

Since the cointegrating vector includes the money supply growth with a negative relation to the nominal interest rate, it is already including a persistent "liquidity" effect of money on the nominal interest rate. In addition the inflation tax effect on the nominal rate is already accounted for within the positive inflation rate term with a Taylor type magnitude of the cointegration coefficient. The stationary error term gives the unexplained leftover from the cointegrating vector. When it is positive, it means the nominal interest rate ends up being higher, although this was not anticipated. Therefore when the money supply growth rate increases are more than the inflation rate increases, the error term is showing that the nominal interest rate is higher in the long run. This can be interpreted as the nominal interest rate rising because the unanticipated money supply growth induces a higher anticipated future inflation rate

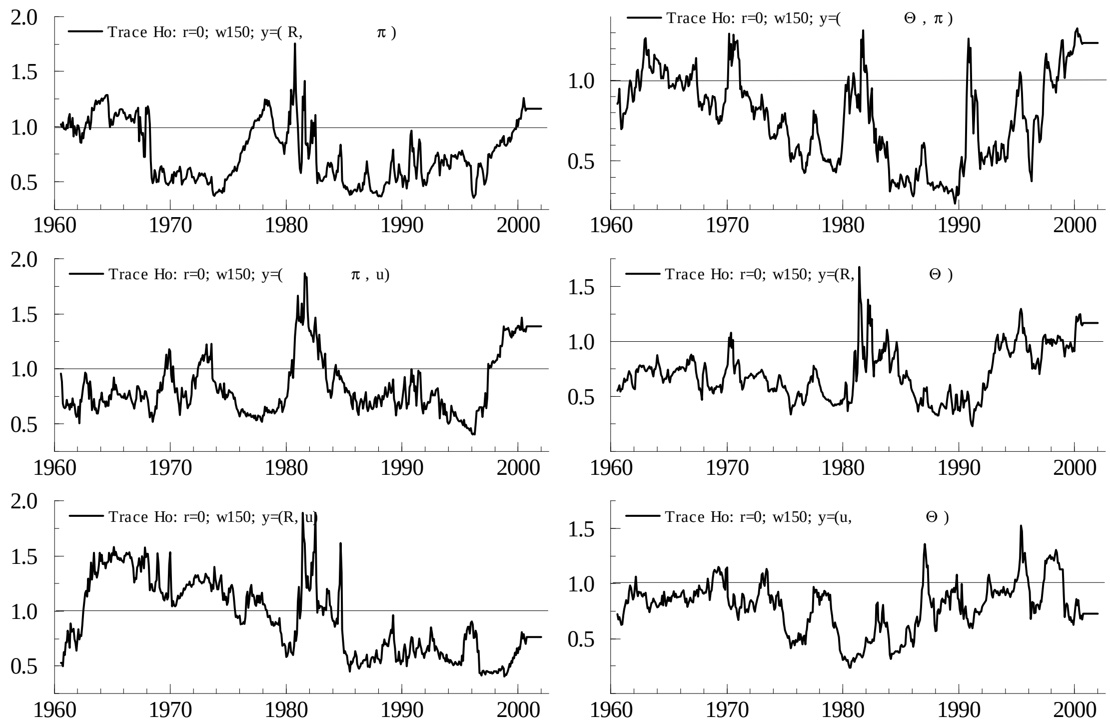


Figure 9: Rolling trace test for all possible pairwise combinations of the four variables Θ, π, R and u.

and so a higher nominal interest rate, since in the long run the money supply growth rate and inflation rate move together. This suggests a residual Fisherian interest rate effect of higher expected inflation from higher money supply growth such that it causes a higher nominal interest rate than what is already accounted for within the cointegrating vector that includes a strong liquidity effect.

For example, the most readily interpretable periods are the lead up to the peak inflation of the early 1980s, and the subsequent rapid decline in the inflation rate. Figure 10 shows that during the lead up to 1980 the money supply growth was less than the inflation rate, just as was the error of the cointegrating vector. This implies that given both the expected liquidity effect from money supply growth rate and the expected inflation effect, on the nominal interest rate, the nominal interest rate was lower than expected as the actual inflation rate outpaced the money supply growth rate. This could be from the liquidity effect of the money supply growth being stronger than expected or from the inflation rate being higher than was expected, both of which could well be expected to occur simultaneously. Note, that similar in spirit to the results in Alvarez and Lippi (2014), the liquidity effect can be expected to be prolonged if the money supply growth rate continues to accelerate for a number of years such as in the late 1970s under President Carter and Fed chairman Miller.

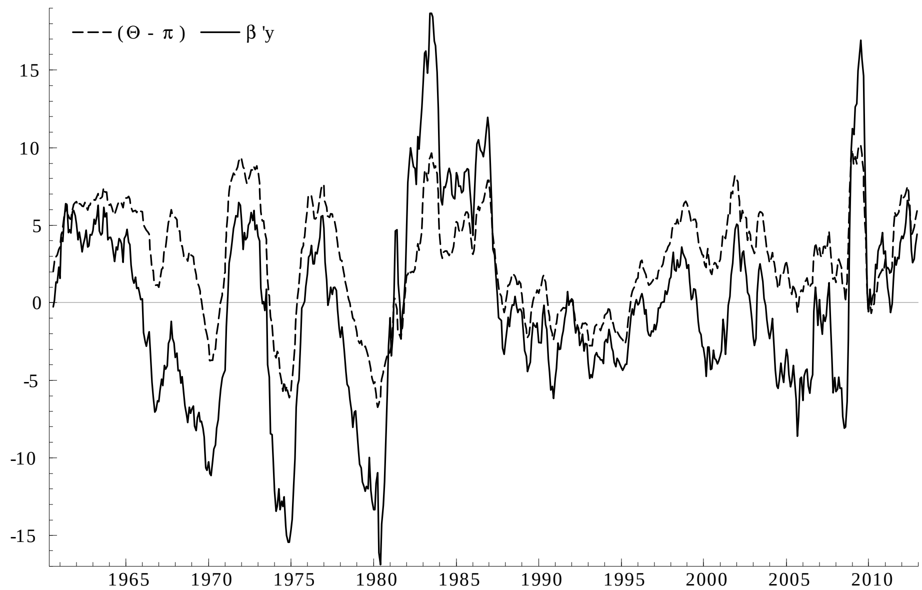
Similarly during the sudden de-acceleration of the money supply growth rate following 1980, the actual money supply growth rate exceeded the inflation rate so as to cause the nominal interest rate to be higher than was predicted by the cointegrating vector. Using similar logic this was due to a prolonged (il-) liquidity effect of the money supply growth rate decrease that was less negative than was anticipated or an inflation rate that was lower than was expected according to the cointegrating relation. Again both are likely to have occurred at once.

This cointegrating relation and the equilibrium error thereby explain the well-known higher nominal interest than that predicted by the Taylor equation before 1980, and the lower nominal interest than is predicted by the Taylor equation after 1980. With this, a completely different explanation from a Taylor reaction function emerges from the equilibrium money supply Euler condition, yet one that is observationally equivalent to the Taylor-type reaction function explanation.

This begs the question as to what the money supply growth rate shocks are being driven by a priori. We prefer to interpret them as the financing needs of the government as it attempts to optimally smooth both fiscal and monetary taxes over time through an inflation targeting strategy from which they must depart during wartime (Vietnam) or bank crisis combined with war. In our model, these occasional fluctuations in the inflation rate are part of the stochastic drift of the money supply growth rate around some bounded mean area as fiscal demands dictate. That monetary policy is viewed as an integral part of fiscal policy, rather than the central bank being some independent entity that does as it wishes, is a dictum of considering the government budget and tax policy including the inflation tax in a unified fashion.

The transition dynamics show ex post how the interest rate gets determined differently in response to such shocks, and this gives rise to our passive versus active phraseology. The inflation tax effect is allowed to dominate the nominal aspect of nominal interest rate changes during expansion, a passive approach that Table 4 shows coincides with relatively low volatilities for all of the dynamic variables. The liquidity money supply effect that the Fed controls more directly dominates the recessionary nominal movements of the nominal interest rate during contractions, a more active approach that Table 4 shows coincides with the highest volatility of the interest rate changes amongst the three regimes. This has intuitive appeal: when things are going well, let them keep going; when things turn down, try more to turn it around with money supply changes. This occurs all at the same time that unemployment is significant in both contraction and expansion with the expected negative sign, and further this unemployment effect is much sharper (of a higher magnitude) during recession than contraction. This is an intuitive way to think of how the real interest falls sharply with unemployment in recession and rises more gradually with less unemployment in expansion.

A related perspective is to view the equilibrium error in terms of Fed Chairmen. Figure 11 shows the graph of the long-run equilibrium error $\beta' y_t^*$ where $\bar{R}_t$ is higher or lower than its equilibrium values in association with the different tenures of Chairmen. It shows that the evolution of the interest rate is more closely determined by the vs EqError



$_1$0.*pdf*

Figure 10: Growth rates of real balances (M2) and the VECM Equilibrium Error Term.

forces underlying the long-run equilibrium relationship during the Greenspan tenure. In contrast, between 1968 and the end of 1987, we can observe large fluctuations in the error, with this period including the Burns-Miller and Volker tenure. The estimated equilibrium error is prevalently positive during the Volker disinflation period and a similar high disequilibrium seems to characterize the more recent period under the Bernanke tenure (i.e. between 2007 and 2012). A discussion in this vein allows an interpretation of whether the Fed Funds rate is "following the Taylor rule or not" (see for e.g. Hayford and Malliaris, 2005), but the Figure 11 actually shows how the economy's equilibrium nominal interest rate diverges from its long run cointegrating relation under different Fed Chairmen.

A qualification of our approach might be viewed to be the empirical plausibility of the money supply growth rate as a unit root process. Statistically, most unit root tests have low power and as long as the series tested to be found as I(1) do not have bubbles then the time series properties of the series do not need to be further constrained. However new approaches are coming online that allow for such further testing such as in Phillips et al. (2013).[24]

Were one econometrically to use structural breaks and reduce the cointegrated series instead into stationary series that could be estimated without cointegration, then the money supply growth rate may no longer be found to be a unit root in postwar US data. It would be stationary with unexplained structural breaks, which one conceivably could be used to define regimes. However looking over the 20th century 30-year filtered US inflation and money supply growth relation in Benati (2009), one might be inclined

[24] We owe this point to Pierre Siklos.

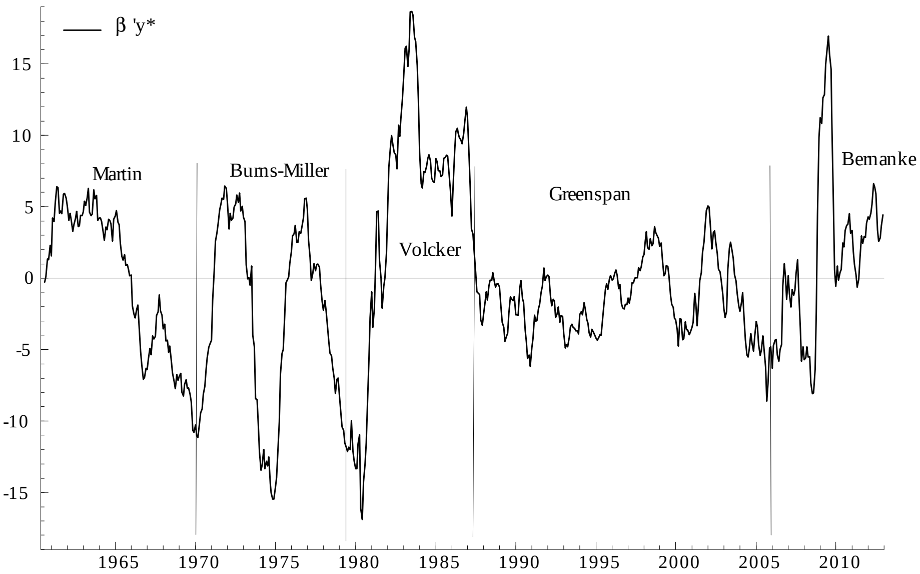


Figure 11: Equilibrium error $\beta' y_t^* = \bar{R}_t - 1.6\bar{\pi}_t + (\Theta_t - \bar{\pi}_t) + 12.2u_t - 23.9$ and US Federal Bank Chairmen's tenures.

instead to characterize the large money supply and inflation movements upwards as war financing through money printing and inflation taxation during WWI, WWII, and the Vietnam war. The Vietnam period appears to give the US post-1960 data its unit root behavior, but that does not mean it should be omitted nor that estimation of the data should ignore a unit root seemingly from occasional, unexpected, war finance.[25]

Widely well-accepted statistical testing methods leave the US postwar money supply growth rate as a unit root, along with the inflation rate, unemployment rate and the nominal interest rate. Embracing this rather than trying to rule it out provides an alternative approach to defining regimes. In particular Markov-switching regimes are well-suited and give novel, alternative regime characterizations for monetary policy in a way consistent with related finance theory findings.

Nonetheless the stability of the money supply growth rate process remains an imperative technically in terms of the existence of the solution of the general equilibrium economy as well as an imperative in Hall and Sargent's (2014) terms for the credibility of long term value of US government debt. This is where Davig and Leeper's (2007) "boundedness" of the money supply process comes in critically. The money supply

[25] The US Congressional Budget Office estimates that the Vietnam war cost a total of 738 billion in 2011 dollars, and involved 2.3% of total US GNP in the peak year of war expenditure, 1968; see Daggett (2010) Costs of Major U.S. Wars, CBO. Also note that the general US Treasury deficit from Vietnam war expenditure was in essence "hidden" by President Johnson's executive action to suddenly put the three Social Security Trust funds on budget in 1969, when they were running large surpluses as the baby boom generation began paying into the system. Historically further back Hall and Sargent (2014) do an insightful study on the three earliest US wars: the Colonial Revolutionary War, War of 1812, and the Civil War. They argue that without substantial revenue otherwise, the Colonial government reverted in essence to printing money to finance the Revolutionary War, In the other two wars, they find that the real and nominal debt remained closely aligned and so inflation finance was relatively less. See also Ohanian (2014).

may experience periods of inflationary finance that cause unit root looking processes. But it still fluctuates over a long period of time around some average stationary money supply growth rate and inflation rate level that in turn keep confidence in the use of the government currency and in the holding of government debt. This boundedness is our shield of the Cochrane (2011) critique about dropping equilibrium assumptions.

In terms of the definition of which money supply to use, we investigated both M1 and M2 aggregates. The use of M2 for the money supply was chosen for the baseline model above as it allows for cointegration. Appendix E sets out the cointegrating vector with M1 instead of M2, and with the restriction as not rejected for a unitary coefficient on the money supply growth. There it is shown that the use of M1 allows for cointegration when dummy breaks are introduced for times of financial deregulation, in particular two breaks in the early 1980s and one over-time break in the early 1990s. These M1 results are not surprising in that they are consistent with a huge literature on finding a lack of cointegration for M1 (Friedman and Kuttner, 1992) because of financial deregulation. For example Friedman and Schwartz (1982) put in dummies for financial deregulation in their study of US-UK velocity. Barnett et al. (1984) have worked around this problem by constructing a pure "money-like" "divisia" monetary aggregate that gives a stable money demand. Alternately some work includes financial sector productivity time series in order to include the price of the money substitute (bank provided exchange credit) within the money demand, resulting in cointegration (Gillman and Otto, 2007), while much recent work isolates such financial sector shocks (Benk et al., 2005, Jermann and Urbana, 2013).

Similar concerns can be expressed about using one particular short run government interest rate such as our baseline use of the Federal Funds rate. Therefore we ran the results also with the 3-month Treasury bill rate. Appendix F presents these results which show little difference from the baseline model with Federal Funds rate, except that we choose a MSIH VECM instead of a MSIAH VECM. The resulting similar cointegrating vector and MS regimes provide additional robustness to the baseline results.

In terms of the interpretation of our novel third regime, we note the similarity to the "lost decade" regime of Gillman et al. (2014) and provide some deeper interpretation here. The recent ambiguity literature for example of Nimark (2014) and Ilut and Schneider (2014) provide intuition on how ambiguity can arise (Nimark) and can play an important role in explaining business cycle periods such as the U.S. recession beginning in 2007 (Ilut and Schneider). Nimark explains how the lack of a signal can induce greater uncertainty by causing the probability of the lower probability event to become assessed with a higher probability of occurring. In application to our monetary model and the associated empirical results, the key signal lost during most of our third "unconventional" regime may be that of the changing nominal interest rate, as a result of the Fed fixing the nominal rate from 2001 to 2004 and since December 2010. This lack of a change in the rate potentially removes a signal to capital markets that results in greater "ambiguity" about financial markets, and the level of the real interest rate and of the inflation rate. Thus in our paper the unconventional period, or "lost decade" regime in Gillman et al., can represent greater ambiguity that induces a type of ex post pessimism that causes the dynamic drift seen in the nominal interest rate and the lack of any variable explaining the unemployment rate, as seen only during this regime.

5 Conclusion

The paper presents evidence of a cointegrated relationship between the nominal interest rate, inflation, the unemployment rate and money growth, for the US 1960-2013 period. The cointegrating equilibrium relationship is characterized by both a stable greater-than-one coefficient for inflation and a liquidity effect from money supply growth. In addition we find a quantity theory type result in that the difference in the money supply growth rate and the inflation rate explain most of the cointegrating vector error term. While the results can be viewed from the perspective of the Taylor rule literature, here our approach is without any explicit reaction function connotations. Rather we present an equilibrium Euler equation combined with an exchange constraint that brings money into the mix that we then estimate. A crucial role is found for the money growth process in determining the nominal interest rate. Dropping the money supply growth rate results in a lack of cointegration as seen in our rolling trace test.

The paper estimates short run dynamic equations with Markov switching regimes. The unemployment rate reduces the nominal interest rate both in the cointegrating vector and in the dynamics for both the contraction and expansion regimes. However the dynamics show no significant effect during the Regime 3's unconventional Fed policy period of late, a time when the unemployment rate has been stressed by policymakers. We also provide an alternative interpretation of active and passive monetary regimes, with the contractions seen as relatively active times and the expansions as relatively passive times.

By adding in the significant money supply growth rate variable, we end up finding support for a Fed policy emphasis on the dual mandate of unemployment and inflation as key factors in nominal interest rate determination.[26] This means that even if the Fed verbally makes the nominal interest rate its target, the money supply growth rate changes in a stable relation with the nominal interest rate along with the inflation rate and unemployment. The error term of the cointegrating relation shows a stable long run one-to-one relation between the money supply growth rate and the inflation rate,

[26] The Employment Act of 1946 (H.R. 50, 95th Congress) mandates that the federal government "promote maximum employment, production, and purchasing power." It was amended in 1978 to the Full Employment and Balanced Growth Act which included specific inflation targets by year, such that within 5 years of the first Economic Report required in the Act, the inflation rate should be three percent; after 1988 the inflation rate should be zero percent. This also means that Volcker could clearly be said to have been trying to implement Congressional policy and US law with his disinflation, in that Congress legislated the target level of the inflation rate by year. Current US law thereby remains that inflation should be zero percent, while the employment rate according to this Act should be three percent or less for those over 20 years of age.

even as the cointegrating vector includes a money supply induced liquidity effect on the interest rate. These money supply effects on interest rates are absent from recent generations of models of employment and interest rates without money. However in this paper we provide robust evidence supporting a theory of interest, money, and the dual mandate.

Or, perhaps Sims and Zha (2006) say it best. While we do not directly address the former part of their following conclusion, we weigh in on the latter part : "neither additive disturbances to a linear monetary policy reaction function nor changes in the coefficients of that function have been a primary source of the rise and fall of inflation over our sample period. Instead, stable monetary policy reactions to a changing array of major disturbances generated the historical pattern."

References

Alvarez, F., Lippi, F., 2014. Persistent Liquidity Effects and Long-Run Money Demand. American Economic Journal: Macroeconomics, 6(2), 71-107.

Alvarez, F., Lucas Jr., R. E., Weber, W. E., 2001. Interest Rate and Inflation. American Economic Review, 91(2), 219-225.

Assenmacher-Wesche, K., 2006. Estimating Central Banks' preferences from a time-varying empirical reaction function. European Economic Review, 50(8), 1951-1974.

Bansal, R., Coleman, W. J., 1996. A Monetary Explanation of the Equity Premium, Term Premium, and Risk-Free Rate Puzzles. Journal of Political Economy, 104(6), 1135-71.

Barnett, W. A., Offenbacher, E. K., Spindt, P. A, 1984. The New Divisia Monetary Aggregates. Journal of Political Economy, 92(6), 1049-85.

Benati, L., 2009. Long Run Evidence on Money Growth and Inflation, ECB Working Paper Series No. 1027, March.

Benk, S., Gillman M., Kejak M., 2005, Credit Shocks in the Financial Deregulatory Era: Not the Usual Suspects. Review of Economic Dynamics, 8(3), 668-687.

Benk, S., Gillman, M., Kejak, M., 2008, Money Velocity in an Endogenous Growth Business Cycle with Credit Shocks. Journal of Money, Credit and Banking, 40(6), 1281-1293.

Benk, S., Gillman M., Kejak M., 2010,. A banking explanation of the US velocity of money: 1919-2004. Journal of Economic Dynamics and Control, 34(4), 765-779.

Bianchi, F., Ilut, C., 2013. Monetary/Fiscal Policy Mix and Agents' Beliefs. Economic Research Initiatives at Duke (ERID), Working Paper No. 119, July.

Bunzel, H., Enders, W., 2005. Is the Taylor rule missing? a statistical investigation. Department of Economics, Iowa State University, Working Paper Series.

Carlson, J. B., Schwarz, J. C., 1999. Effects of movements in equities prices on M2 demand. Economic Review-Federal Reserve Bank of Cleveland, 35(4): 2-9.

Castelnuovo, E., Greco, L., Raggi, D., 2012. Policy rules, regime switches, and trend inflation: an empirical investigation for the United States. Macroeconomic Dynamics, 1(1), 1-23.

Clark, J. A., 1984. Estimation of Economies of Scale in Banking Using a Generalized Functional Form. Journal of Money, Credit and Banking, 16(1), 53-68.

Cochrane, J. H. 2011. Determinacy and Identification with Taylor Rules. Journal of Political Economy, 119(3), 565 - 615.

Cogley, T., Sbordone, A. M., 2005. A Search for a Structural Phillips Curve. Staff Report 203. New York: Federal Reserve Bank of New York.

Cooley, T., Hansen, G., 1989. The inflation tax in a real business cycle model. American Economic Review, 79(4), 733-748.

Daggett, S., 2010. Costs of Major U.S. Wars, Congressional Budget Office, June 29.

Davies, C., Gillman M., Kejak M., 2012. Deriving the Taylor Principle when the Central Bank Supplies Money. Economics Working Paper E2012/20, Cardiff University.

Davies, R. B., 1987. Hypothesis testing when a nuisance parameter is present only under the alternative. Biometrika, 64, 247-254.

Davig T., Leeper E. M., 2007. Generalizing the Taylor Principle. American Economic Review, 97(3), 607-635.

Doornik, J. A., 1998. Approximations to the asymptotic distribution of cointegration tests. Journal of Economic Surveys, 12(5), 573-593.

Elliott, G., Rothenberg, T., & Stock, J. H., 1996. Efficient tests for an autoregressive unit root. Econometrica, 64, 813-836.

Engle, R. F., Granger, C. W., 1987. Co-integration and error correction: representation, estimation, and testing. Econometrica, 55(2), 251-276.

Erceg, C., Levin, J., Andrew, T., 2003. Imperfect credibility and inflation persistence, Journal of Monetary Economics, 50(4), 915-944.

Francis, N., Owyang, M., T., 2005. Monetary Policy in a Markov-Switching VECM: Implications for the Cost of Disinflation and the Price Puzzle. Journal of Business and Economic Statistics, 23(3), 305-13.

Friedman, B. M., Kuttner, K. N., 1992. Money, Income, Prices, and Interest Rates. American Economic Review, 82(3), 472-492.

Friedman, Milton. 1969. The optimum quantity of money, and other essays; Chicago: Aldine Pub. Co.

Friedman, Milton, and Schwartz, Anna J., 1982, Monetary Trends in the United States and the United Kingdom, The University of Chicago Press, Chicago.

Garcia, R., 1998. Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models. International Economic Review, 39, 763-788.

Gavin, W. T., Keen, B. D., Pakko, M. R., 2005. The Monetary Instrument Matters. Federal Reserve Bank of St. Louis Review, 87, 633-58.

Gerlach-Kristen, P., 2003. Interest rate reaction functions and the Taylor rule in the Euro area. Working Paper Series 0258, European Central Bank.

Gillman, M., Kejak, M., Pakos, M., 2014. Learning about Disaster Risk: Joint Implications for Stocks and Bonds, forthcoming, Review of Finance.

Gillman, M., Otto, G., 2007. Money Demand in General Equilibrium Endogenous Growth: Estimating the Role of a Variable Interest Elasticity. Quantitative and Qualitative Analysis in Social Sciences, 1(1), 1-25.

Gomme, P., 1993. Money and growth revisited: Measuring the costs of inflation in an endogenous growth model. Journal of Monetary Economics, 32(1), 51-77.

Goodfriend, M., 1993. Interest rate policy and the inflation scare problem: 1979–1992. Economic Quarterly, Federal Reserve Bank of Richmond, 79(1), 1-24.

Goodfriend, M., King, R. G., 2005. The incredible Volcker disinflation. Journal of Monetary Economics, 52, 981-1015.

Granger, C. W., Newbold, P., 1974. Spurious regressions in econometrics. Journal of Econometrics, 2(2), 111-120.

Hall, G. J., Sargent, T., J., 2014. Fiscal discriminations in three wars. Journal of Monetary Economics, 61, 148-166.

Hamilton, J. D., 1989. A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. Econometrica, 57, 357-384.

Hancock, D.,1985. The financial firm: production with monetary and nonmonetary goods. Journal of Political Economy, 93(5), 859-880.

Hansen, B.E., 1992. The Likelihood Ratio Test under Nonstandard Conditions: Testing the Markov Switching Model of GNP. Journal of Applied Econometrics, 7, S61-S82.

Hansen, B.E., 1996. Erratum: The Likelihood Ratio Test under Nonstandard Conditions: Testing the Markov Switching Model of GNP. Journal of Applied Econometrics, 11, 195-198.

Harding, Don and Pagan, Adrian, 2002. "Dissecting the cycle: a methodological investigation," Journal of Monetary Economics, 49 (2, March): 365-381.

Hansen, P., Johansen S., 1998. Workbook on Cointegration. Oxford: Oxford University Press.

Hayford, M. D., Malliaris, A. G., 2005. How did the Fed react to the 1990s stock market bubble? Evidence from an extended Taylor rule. European Journal of Operational Research, 163(1), 20-29.

Hicks, J. R., 1935. A Suggestion for Simplifying the Theory of Money, Economica, 2, 1-19.

Humphrey, D. B., and Berger, A. N., 1997. Efficiency of Financial Institutions: International Survey and Directions for Future Research. European Journal of Operations Research, 98(2), 175-212.

Ilut, C. L., and Schneider, M. 2014. Ambiguous Business Cycles. American Economic Review, 104 (8), 2368-99.

Ireland, P. N., 2007. Changes in the Federal Reserve's inflation target: Causes and consequences. Journal of Money, Credit and Banking, 39(8), 1851-1882.

Jermann, U., Quadrini, V., 2012. Macroeconomic Effects of Financial Shocks. American Economic Review, 102(1), 238-71.

Johansen, S., 1988. Statistical analysis of cointegrating vectors. Journal of Economic Dynamics and Control, 12(2), 231-254.

Johansen, S., 1991. Estimation and Hypothesis Testing of Cointegrating Vectors in Gaussian Vector Autoregressive Models. Econometrica, 59, 1551-1580.

Johansen, S., 2006. Confronting the economic model with the data. In: Colander, D. ed. Post Walrasian Macroeconomics. Beyond the Dynamic Stochastic General Equilibrium Model. Cambridge University Press: Cambridge, pp. 287–300.

Krolzig, H-M., 1997. Markov-Switching Vector Autoregressions, New York: Springer.

Krolzig, H-M., 1998. Econometric modeling of Markov-switching vector autoregressions using MSVAR for Ox, Discussion Paper, Institute of Economics and Statistics, Oxford.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., & Shin, Y., 1992. Testing the null hypothesis of stationary against the alternative of a unit root. Journal of Econometrics, 54(1), 159-178.

Leeper, E. M., Roush, J. E., 2003. Putting "M" back in monetary policy. Journal of Money, Credit and Banking, 35(6), 1217-1256.

Leeper, E. M., Zha, T., 2003. Modest policy interventions. Journal of Monetary Economics, 50(8), 1673-1700.

Lucas, Jr., R. E., Stokey, N. L., 1983. Optimal fiscal and monetary policy in an economy without capital. Journal of Monetary Economics, 12(1), 55-93.

MacKinnon, J.G., Haug, A.A., Michelis, L., 1999. Numerical distribution functions of likelihood ratio tests for cointegration. Journal of Applied Econometrics, 14, 563–577.

McCallum, B. T., 1990. Could a monetary base rule have prevented the great depression?. Journal of Monetary Economics, 26(1), 3-26.

McCallum, B. T., 1999. Recent developments in the analysis of monetary policy rules, Review, Federal Reserve Bank of St. Louis, November, 3-12.

Mehra, Y. P., 1997. A review of the recent behavior of M2 demand. Economic Quarterly-Federal Reserve Bank of Richmond, 83, 27-44.

Minford, P., Perugini, F., Srinivasan, N., 2002. Are interest rate regressions evidence for a Taylor rule?. Economics Letters, 76(1), 145-150.

Nimark, Kristoffer P., 2014. Man-Bites-Dog Business Cycles. American Economic Review, 104 (8), 2320-67.

Ohanian, L. E., 2014. Comment on: “Fiscal discriminations in three wars” by Hall and Sargent. Journal of Monetary Economics, 61,167-171.

Österholm, P., 2005. The Taylor Rule: A Spurious Regression?. Bulletin of Economic Research, 57(3), 217-247.

Orphanides, A., 2003. Historical monetary policy analysis and the Taylor rule. Journal of Monetary Economics, 50(5), 983-1022.

Phillips, P. C., 1986. Understanding spurious regressions in econometrics. Journal of econometrics, 33(3), 311-340.

Phillips, P. C. B., Shi, Shu-Ping, Yu, J., 2013. Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P 500. Cowles Foundation Discussion Papers 1914, Cowles Foundation for Research in Economics, Yale University.

Rangvid, J., Sørensen, C., 2002. Convergence in the ERM and declining numbers of common stochastic trends. Journal of Emerging Market Finance, 1(2), 183-213.

Roberts, J. M., 2006. Monetary Policy and Inflation Dynamics. International Journal of Central Banking, 2, 193–230.

Saikkonen, P., 1992. Estimation and testing of cointegrated systems by an autoregressive approximation. Econometric Theory, 8(01), 1-27.

Saikkonen, P., Luukkonen, R., 1997. Testing cointegration in infinite order vector autoregressive processes. Journal of Econometrics, 81, 93-126.

Salemi, M. K., 2006. Econometric Policy Evaluation and Inverse Control. Journal of Money, Credit, and Banking, 38, 1737–64.

Sarno, L., Valente, G., 2000. The Cost of Carry Model and Regime Shifts in Stock Index Ritures Markets:

An Empirical Investigation. Journal of Futures Markets, 20, 603-624.

Sarno, L., Valente, G., Wohar, M. E., 2004. Monetary fundamentals and exchange rate dynamics under different nominal regimes. Economic Inquiry, 42(2), 179-193.

Schabert, A., 2003. On the Equivalence of Money Growth and Interest Rate Policy. Working Paper 2003-6, Department of Economics, University of Glasgow, April.

Siklos, P. L., Wohar M. E., 2006. Estimating Taylor-type Rules: An Unbalanced Regression?. Advances in Econometrics, 20, 239-276.

Sims, C. A., Zha, T., 2002. Macroeconomic switching. manuscript, Princeton University.

Sims, C. A., Zha, T., 2006. Were there regime switches in US monetary policy?. The American Economic Review, 96(1), 54-81.

Smets, F., Wouters, R., 2003. An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. Journal of the European Economic Association, 1(5): 1123-1175.

Smets, F., Wouters, R., 2007. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. The American Economic Review, 97(3), 586-606.

Stock, J. H., 1987. Asymptotic properties of least squares estimators of cointegrating vectors. Econometrica, 55, 1035-1056.

Taylor, J. B., 1993. Discretion versus Policy Rules in Practice. Carnegie-Rochester Conference Series on Public Policy, 39(1), 195-214.

Taylor, J.B., 1999. An historical analysis of monetary policy rules. In: Taylor, J.B. (Ed.), Monetary Policy Rules. University of Chicago Press for NBER, Chicago.

Thornton, D. L., 2014. Monetary Policy: Why Money Matters (and Interest Rates Don't). Journal of Macroeconomics, forthcoming.

Trecroci, C., Vassalli, M., 2010. Monetary policy regime shifts: new evidence from time-varying interest rate rules. Economic Inquiry, 48(4), 933-950.

Valente, G., 2003a. Monetary Policy Rules and Regime Shifts. Applied Financial Economics, 13(7), 525–535.

Valente, G., 2003b. Essays in financial forecasting. Doctoral dissertation, University of Warwick.

Woodford, M., 2008. How important is money in the conduct of monetary policy?. Journal of Money, Credit and Banking, 40(8), 1561-1598.

Online Appendix A: ADF, DF-GLS and KPSS Tests

Table 1A shows the results of the ADF tests, DF-GLS tests (Elliott *et al.*, 1996) and KPSS tests (Kwiatkowski *et al.*, 1992), allowing for an intercept, as the deterministic component, in the level of g_c. The column denoted Lags reports the maximum lag, which was selected on the basis of the Akaike information criterion (AIC) and also chosen in order to avoid autocorrelated residuals of each ADF regression. All the results show the presence of a unit root in levels of the variables, since we were unable to reject the unit root null hypothesis at conventional levels of significance, and KPSS stationarity tests confirm this result. Panel b of Table 1 shows that differencing the series induce stationarity in each case without ambiguity. Therefore, we conclude that the examined series are a realization from a stochastic process integrated of order one I(1).

Table 1A - *Unit-root test*

Panel a: Variables in levels				
$Variables$	$Lags$	ADF	$DFGLS$	$KPSS$
$\overline{R}$	13	-2.426	-1.908	1.259
$\overline{\pi}$	13	-2.186	-1.729	0.876
Θ	12	-2.291	-1.367	1.026
u	3	-2.192	-1.925	1.388
Panel b: Variables in differences				
$Variables$	$Lags$	ADF	$DFGLS$	$KPSS$
$\Delta\overline{R}$	13	-6.270	-4.765	0.089
$\Delta\overline{\pi}$	13	-7.245	-5.204	0.067
$\Delta\Theta$	12	-8.375	-4.923	0.030
Δu	3	-8.029	-3.737	0.097

Note. Critical values at the 5 and 1 percent significance levels for the ADF test for the unit root null, in the case of a constant in the regression, are -2.87, -3.44, respectively. Critical values at the 10, 5 and 1 percent significance levels for the DF-GLS test (Elliott et al., 1996) for the unit root null, in the case of a constant as the deterministic component of the regression, are -2.62, -2.03 and -1.73, respectively. The column denoted Lags reports the maximum lag, which was selected on the basis of the Akaike Information Criterion (AIC) and to avoid autocorrelated residuals of each ADF regression. Critical values at the 10, 5 and 1 percent significance levels for the KPSS test (Kwiatkowski et al., 1992) for the null of stationarity, in the case of a constant as the deterministic component of the regression, are 0.35, 0.46 and 0.74, respectively.

A highly persistent series, with a root very near to unity, is in practice indistinguishable from a true unit root and it is better approximated by I(1) process than by stationary ones (while acknowledging the alternative of fractional cointegration). Moreover, as shown by Johansen (2006), the cost of treating near unit roots as stationary is that the standard asymptotic distributions provide very poor approximations to the finite sample distributions of the estimated steady-state values.

Online APPENDIX B: Data Analysis

First is a data description in Table 1B and graphs of differenced data in Figure 1B. The stationarity of the change in the variables entering the cointegrating vector can be seen in Appendix A. The stationarity of the change in the variables entering the cointegrating vector can be seen in both Figure 1B and Table 1B.

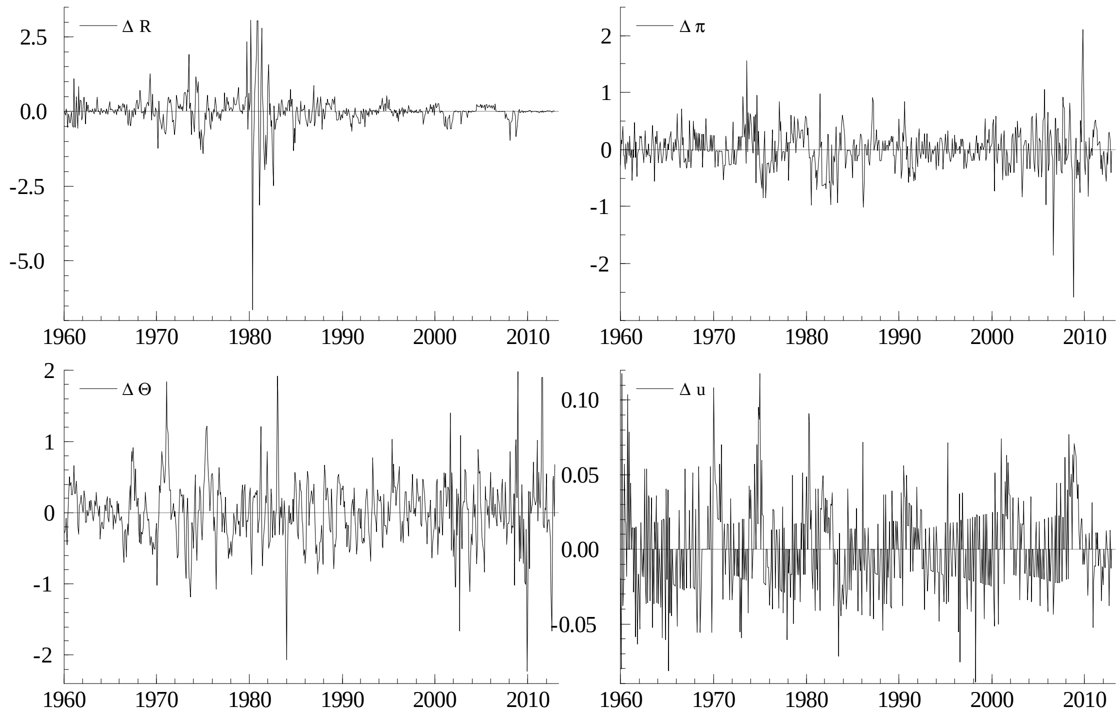


Figure1B: Differenced data. The sample is 1960.1 2012.12.

Table 1B - *Statistics and normality test*

Mean	Max	Min	Std. Dev	Skewness	Kurtosis	Normality test $\chi^2(2)$
5.5659	19.1	0.07	3.5490	0.9387	1.5279	87.080[0.0000] $**$
3.9202	13.6	-2.0	2.7162	1.4034	1.9223	374.73[0.0000] $**$
6.6964	12.9	0.21	2.7405	-0.0069	-0.1194	0.1771[0.9153]
1.7725	2.38	1.22	0.2619	0.07433	-0.5077	8.4066[0.0149]$*$

Next is choosing the congruent VAR specification. Starting with a VAR(7), we first conduct tests of model reduction within a framework of nested models specification. On the basis of the AIC information criteria we choose a VAR(6). On the contrary, the SC and HQ information criteria choose a more parsimonious formulation, like a VAR(2) (see Table2aB). Therefore, in order to choose between the VAR(6) and VAR(2) parameterization, we adopt also the F-test on a group of coefficients: Table 3aB shows that the only reduction which is not rejected is that from a VAR(7) to VAR(6); all other reductions are rejected and we observe that a VAR(2) is never accepted if tested against all the other lags.

Moreover, we conduct the LM test on autocorrelation both for a VAR(6) and a VAR(2) specification and we see that there is no autocorrelation of order 1 and 6 in the VAR(6), but there is autocorrelation of order 1 and 2 in the VAR(2). Undertaking the same model selection procedure, but starting from a maximum lag of nine periods, we reach the same conclusion (see Tables 2bB and 3bB).

More importantly, we find a confirmation of our choice also if we check for autocorrelation, which is the major concern in the choice of the congruent VAR. Table 4B reports the tests for autocorrelation, respectively, of order 6 and order 1 in a VAR(6) specification: the first row of Panel a reports the test for the system, while the other rows report the autocorrelation tests in each single equation of the system. Table 5B does the same in the VAR(2) specification.

Table 2aB

Model	T	n		log-likelihood	SC	HQ	AIC
VAR(7)	629	116	OLS	551.18073	−0.56414	−1.0654	−1.3837
VAR(6)	**629**	**100**	**OLS**	**540.55244**	−0.69426	−1.1263	**−1.4008**
VAR(5)	629	84	OLS	522.22564	−0.79991	−1.1629	−1.3934
VAR(4)	629	68	OLS	497.15841	−0.88413	−1.1779	−1.3646
VAR(3)	629	52	OLS	474.41971	−0.97575	−1.2004	−1.3431
VAR(2)	**629**	**36**	**OLS**	**445.22451**	**−1.0468**	**−1.2024**	−1.3012
VAR(1)	629	20	OLS	210.36027	−0.46397	−0.55039	−0.60528

Table 3aB - *Tests of model reduction (models are nested for test validity)*

VAR(7)->VAR(6):F(16,1824)=1.2684[0.2088]
VAR(7)->VAR(5):F(32,2203)=1.7405[0.0063]**
VAR(7)->VAR(4):F(48,2301)=2.1867[0.0000]**
VAR(7)->VAR(3):F(64,2339)=2.3515[0.0000]**
VAR(7)->VAR(2):F(80,2357)=2.6273[0.0000]**
VAR(7)->VAR(1):F(96,2367)=7.7594[0.0000]**
VAR(6)->VAR(5):F(16,1836)=2.2106[0.0038]**
VAR(6)->VAR(4):F(32,2217)=2.6424[0.0000]**
VAR(6)->VAR(3):F(48,2317)=2.7084[0.0000]**
VAR(6)->VAR(2):F(64,2355)=2.9623[0.0000]**
VAR(6)->VAR(1):F(80,2373)=9.0458[0.0000]**
VAR(5)->VAR(4):F(16,1848)=3.0546[0.0000]**
VAR(5)->VAR(3):F(32,2232)=2.9360[0.0000]**
VAR(5)->VAR(2):F(48,2332)=3.1889[0.0000]**
VAR(5)->VAR(1):F(64,2370)=10.678[0.0000]**
VAR(4)->VAR(3):F(16,1861)=2.7857[0.0002]**
VAR(4)->VAR(2):F(32,2247)=3.2164[0.0000]**
VAR(4)->VAR(1):F(48,2347)=13.066[0.0000]**
VAR(3)->VAR(2):F(16,1873)=3.6124[0.0000]**
VAR(3)->VAR(1):F(32,2262)=18.075[0.0000]**
VAR(2)->VAR(1):F(16,1885)=32.634[0.0000]**

Table 3bB - *Tests of model reduction (models are nested for test validity)*

VAR(9) –> VAR(8): F(16,1793)= 3.1086 [0.0000]**
VAR(9) –> VAR(7): F(32,2166)= 2.4130 [0.0000]**
VAR(9) –> VAR(6): F(48,2263)= 2.0490 [0.0000]**
VAR(9) –> VAR(5): F(64,2300)= 2.0949 [0.0000]**
VAR(9) –> VAR(4): F(80,2318)= 2.3015 [0.0000]**
VAR(9) –> VAR(3): F(96,2327)= 2.4076 [0.0000]**
VAR(9) –> VAR(2): F(112,2333)= 2.6047 [0.0000]**
VAR(9) –> VAR(1): F(128,2337)= 6.5150 [0.0000]**
VAR(8) –> VAR(7): F(16,1806)= 1.6980 [0.0407]*
VAR(8) –> VAR(6): F(32,2181)= 1.5007 [0.0357]*
VAR(8) –> VAR(5): F(48,2278)= 1.7344 [0.0014]**
VAR(8) –> VAR(4): F(64,2315)= 2.0718 [0.0000]**
VAR(8) –> VAR(3): F(80,2333)= 2.2369 [0.0000]**
VAR(8) –> VAR(2): F(96,2343)= 2.4865 [0.0000]**
VAR(8) –> VAR(1): F(112,2349)= 6.9054 [0.0000]**
VAR(7) –> VAR(6): F(16,1818)= 1.2984 [0.1889]
VAR(7) –> VAR(5): F(32,2195)= 1.7453 [0.0061]**
VAR(7) –> VAR(4): F(48,2294)= 2.1869 [0.0000]**
VAR(7) –> VAR(3): F(64,2331)= 2.3611 [0.0000]**
VAR(7) –> VAR(2): F(80,2349)= 2.6323 [0.0000]**
VAR(7) –> VAR(1): F(96,2359)= 7.7394 [0.0000]**
VAR(6) –> VAR(5): F(16,1830)= 2.1897 [0.0042]**
VAR(6) –> VAR(4): F(32,2210)= 2.6271 [0.0000]**
VAR(6) –> VAR(3): F(48,2309)= 2.7107 [0.0000]**
VAR(6) –> VAR(2): F(64,2347)= 2.9605 [0.0000]**
VAR(6) –> VAR(1): F(80,2365)= 9.0141 [0.0000]**
VAR(5) –> VAR(4): F(16,1842)= 3.0453 [0.0000]**
VAR(5) –> VAR(3): F(32,2225)= 2.9501 [0.0000]**
VAR(5) –> VAR(2): F(48,2324)= 3.1937 [0.0000]**
VAR(5) –> VAR(1): F(64,2362)= 10.645 [0.0000]**
VAR(4) –> VAR(3): F(16,1855)= 2.8227 [0.0001]**
VAR(4) –> VAR(2): F(32,2240)= 3.2281 [0.0000]**
VAR(4) –> VAR(1): F(48,2340)= 13.025 [0.0000]**
VAR(3) –> VAR(2): F(16,1867)= 3.5980 [0.0000]**
VAR(3) –> VAR(1): F(32,2254)= 17.991 [0.0000]**
VAR(2) –> VAR(1): F(16,1879)= 32.481 [0.0000]**

Table 4B - *Testing for error autocorrelation in VAR(6)*

Panel a: Testing for Vector error autocorrelation from lags 1 to 6
$\chi^2(96)$= 173.64 [0.0000]** and F-form F(96,2288)= 1.7926 [0.0000]**
$\bar{R}_t$: AR 1-6 test: F(6,598) = 1.5800 [0.1504]
$\bar{\pi}_t$: AR 1-6 test: F(6,598) = 1.6038 [0.1436]
Θ_t: AR 1-6 test: F(6,598) = 1.0429 [0.3962]
u_t: AR 1-6 test: F(6,598) = 1.3518 [0.2321]
Panel b: Testing for Vector error autocorrelation from lags 1 to 1
$\chi^2(16)$= 14.074 [0.5932] and F-form F(16,1824)= 0.84205 [0.6378]
$\bar{R}_t$: AR 1-1 test: F(1,603) = 0.3156 [0.5745]
$\bar{\pi}_t$: AR 1-1 test: F(1,603) = 0.0512 [0.8211]
Θ_t: AR 1-1 test: F(1,603) = 1.3351 [0.2484]
u_t: AR 1-1 test: F(1,603) = 0.8565 [0.3551]

Table 5B - *Testing for error autocorrelation in VAR(2)*

Panel a: Testing for Vector error autocorrelation from lags 1 to 2
$\chi^2(32)$=95.089 [0.0000]** and F-form F(32,2247)=3.0202 [0.0000]**
$\bar{R}_t$: AR 1-2 test: F(2,618) = 5.4561 [0.0045]**
$\bar{\pi}_t$: AR 1-2 test: F(2,618) = 0.0111 [0.9889]
Θ_t : AR 1-2 test: F(2,618) = 2.1549 [0.1168]
u_t : AR 1-2 test: F(2,618) = 10.6740 [0.0000]**
Panel b: Testing for Vector error autocorrelation from lags 1 to 1
$\chi^2(32)$=95.089 [0.0000]** and F-form F(32,2247)=3.0202 [0.0000]**
$\bar{R}_t$: AR 1-1 test: F(1,619) = 6.1284 [0.0136]*
$\bar{\pi}_t$: AR 1-1 test: F(1,619) = 0.0216 [0.8832]
Θ_t : AR 1-1 test: F(1,619) = 2.1777 [0.1405]
u_t : AR 1-1 test: F(1,619) = 1.8212 [0.1777]

Online APPENDIX C: Markov Switching Lags and Regimes

Table 1C reports all the model selection criteria in the MSIAH framework. More specifically, in the case of two regimes it is difficult to choose the maximum lag, since there is not a coherent indication given by the information criteria. The AIC criterion tends to over-parameterize, but the HQ and SC choose a VECM(1). In the case of three regimes a MSIAH(3)-VECM(1) is preferred, and this model also dominates the two-regimes version. This means that, although the estimation of three regimes increases the number of parameters, it dominates the two-regimes model, since the improvement in the likelihood outweighs the cost of estimating a model with a greater number of parameters and this is indicated by all the information criteria for more than one lag (e. g. AIC clearly prefers the three-regimes model even up to the fifth lag). Tables 1aC and 1bC report all other information in terms of probability and duration of regimes, respectively, in the two-state and three-state Markov switching estimation.

Table 1C - *Model selection criteria in the MSIAH-VECM framework*

Model	LR	Information Criteria (IC)			n
MSIAH(M)-VECM(k-1)	Log-likelihood	AIC	HQ	SC	
MSIAH(2)-VECM(5)	1005.7386	−2.5683	−2.0249	−1.1694	198
MSIAH(2)-VECM(4)	972.7061	−2.5650	−2.1094	−1.3922	166
MSIAH(2)-VECM(3)	953.4520	**−2.6056**	−2.2378	−1.6588	134
MSIAH(2)-VECM(2)	921.1298	−2.6045	−2.3246	−1.8839	102
MSIAH(2)-VECM(1)	880.2042	−2.5762	**−2.3840**	**−2.0816**	70
MSIAH(3)-VECM(5)	1133.9300	−2.6516	−1.8282	−0.5320	300
MSIAH(3)-VECM(4)	1088.8023	−2.6607	−1.969	−0.8803	252
MSIAH(3)-VECM(3)	1096.3686	−2.8374	−2.2775	−1.3961	204
MSIAH(3)-VECM(2)	1125.5045	−3.0827	−2.6545	−1.9805	156
MSIAH (3)-VECM(1)	**1090.2598**	**−3.1232**	**−2.8268**	**−2.3602**	**108**

n is the number of parameters and k is the maximum lag in VAR specification

Table 1aC - *Regime properties of MSIAH(2)-VECM(k-1)*

MSIAH(M)-VECM(k-1)	p_{11}	p_{12}	duration 1	duration 2
MSIAH(2)-VECM(5)	0.81	0.95	5.29	19.63
MSIAH(2)-VECM(4)	0.81	0.95	5.15	22.15
MSIAH(2)-VECM(3)	0.89	0.97	8.71	37.12
MSIAH(2)-VECM(2)	0.86	0.97	7.36	33.06
MSIAH(2)-VECM(1)	0.86	0.97	6.98	33.93

p_{ii} denote the transition probabilities obtained from the Markov-switching model, "duration i" denotes the expected duration (in months) of each regime i.

Table 1bC - *Regime properties of MSAH(3)-VECM(k-1)*

MSAH(M)-VECM(k-1)	p_{11}	p_{22}	p_{33}	duration regime 1	duration regime 2	duration regime 3
MSAH(3)-VECM(5)	0.7726	0.9332	0.8553	4.40	14.96	6.91
MSAH(3)-VECM(4)	0.8322	0.9485	0.9372	5.96	19.42	15.92
MSAH(3)-VECM(3)	0.8673	0.9186	0.7988	7.54	12.29	4.97
MSAH(3)-VECM(2)	0.8635	0.9301	0.8450	7.32	14.31	6.45
MSAH(3)-VECM(1)	0.8957	0.9582	0.9152	9.58	23.91	11.79

p_{ii} denote the transition probabilities obtained from the Markov-switching model, "duration regime i" denotes the expected duration (in months) of each regime i.

Table 2C reports all the model selection criteria in the alternative MSAH framework. Observations done so far for the MSIAH model are valid also in this contest and the preferred model is a MSAH(3)-VECM(1). Tables 2aC and 2bC report all other information in terms of probability and duration of regimes, respectively, in the two-state and three-state Markov switching estimation. Although the conclusions are the same, we report also all correspondent tables for the MSH(M)-VECM(k-1) specification; see Tables 3C, 3aC and 3bC.

It must be stressed that we have also estimated other versions of Markov-Switching VECM, but for space considerations, we report only the three versions which are interesting to the analysis. Here we just want to observe that the dominant aspect in this model selection procedure is a clear improvement when introducing the shift in the variance-covariance matrix.

Table 2C - *Model selection criteria in the MSAH-VECM framework*

Model	LR	Information Criteria			n
MSAH(M)-VECM(k-1)	Log-likelihood	AIC	HQ	SC	
MSAH(2)-VECM(5)	1006.9289	−2.5848	−2.0524	−1.2141	194
MSAH(2)-VECM(4)	970.8371	−2.5718	−2.1272	−1.4272	162
MSAH(2)-VECM(3)	963.4160	**−2.6500**	−2.2932	−1.7315	130
MSAH(2)-VECM(2)	924.2287	−2.6271	−2.3581	−1.9347	98
MSAH(2)-VECM(1)	886.8219	−2.6099	**−2.4288**	**−2.1436**	66
MSAH(3)-VECM(5)	1166.1133	−2.7794	−1.9780	−0.7163	292
MSAH(3)-VECM(4)	1159.3932	−2.9106	−2.2410	−1.1867	244
MSAH(3)-VECM(3)	1146.6899	−3.0229	−2.4849	−1.6380	196
MSAH(3)-VECM(2)	1112.0541	−3.0654	−2.6592	−2.0197	148
MSAH (3)-VECM(1)	**1078.4998**	**−3.1113**	**−2.8368**	**−2.4047**	**100**

n is the number of parameters and k is the maximum lag in VAR specification

Table 2aC - *Regime properties of MSAH(2)-VECM(k-1)*

MSAH(M)-VECM(k-1)	p_{11}	p_{12}	duration 1	duration 2
MSAH(2)-VECM(5)	0.9428	0.7931	17.47	4.83
MSAH(2)-VECM(4)	0.9393	0.7303	16.47	3.71
MSAH(2)-VECM(3)	0.9431	0.7988	17.56	4.97
MSAH(2)-VECM(2)	0.9623	0.8176	26.56	5.48
MSAH(2)-VECM(1)	0.9670	0.8585	30.27	7.07

p_{ii} denote the transition probabilities obtained from the Markov-switching model, "duration i" denotes the expected duration (in months) of each regime i.

Table 2bC - *Regime properties of MSAH(3)-VECM(k-1)*

				duration		
MSAH(M)-VECM(k-1)	p_{11}	p_{12}	p_{13}	regime 1	regime 2	regime 3
MSAH(3)-VECM(5)	0.7726	0.9332	0.8553	4.40	14.96	6.91
MSAH(3)-VECM(4)	0.8322	0.9485	0.9372	5.96	19.42	15.92
MSAH(3)-VECM(3)	0.8673	0.9186	0.7988	7.54	12.29	4.97
MSAH(3)-VECM(2)	0.8635	0.9301	0.8450	7.32	14.31	6.45
MSAH(3)-VECM(1)	0.8957	0.9582	0.9152	9.58	23.91	11.79

p_{ii} denote the transition probabilities obtained from the Markov-switching model, "duration regime i" denotes the expected duration (in months) of each regime i.

Table 3C - *Model selection criteria in the MSH-VECM framework*

Model	LR	Information Criteria			n
MSH(M)-VECM(k-1)	Log-likelihood	AIC	HQ	SC	
MSH(2)-VECM(5)	925.1765	-2.5920	-2.2901	-1.8148	110
MSH(2)-VECM(4)	910.5296	-2.5963	-2.3383	-1.9321	94
MSH(2)-VECM(3)	889.6734	$\mathbf{-2.5808}$	-2.3668	-2.0297	78
MSH(2)-VECM(2)	869.0603	-2.5662	-2.3960	-2.1281	62
MSH(2)-VECM(1)	840.1273	-2.5250	$\mathbf{-2.3988}$	$\mathbf{-2.2000}$	46
MSH(3)-VECM(5)	1059.4854	-2.9745	-2.6342	-2.0984	124
MSH(3)-VECM(4)	1050.5644	-2.9970	-2.7006	-2.2340	108
MSH(3)-VECM(3)	1036.0713	$\mathbf{-3.0018}$	-2.7493	-2.3518	92
MSH(3)-VECM(2)	1006.6770	-2.9592	-2.7506	-2.4223	76
MSH (3)-VECM(1)	991.9332	-2.9632	$\mathbf{-2.7985}$	$\mathbf{-2.5393}$	60

n is the number of parameters and k is the maximum lag in VAR specification

Table 3aC - *Regime properties of MSH(2)-VECM(k-1)*

MSH(M)-VECM(k-1)	p_{11}	p_{22}	duration 1	duration 2
MSH(2)-VECM(5)	0.8265	0.9543	5.76	21.89
MSH(2)-VECM(4)	0.8247	0.9540	5.70	21.76
MSH(2)-VECM(3)	0.8288	0.9550	5.84	22.21
MSH(2)-VECM(2)	0.8261	0.9577	5.75	23.64
MSH(2)-VECM(1)	0.8224	0.9556	5.63	22.51

p_{ii} denote the transition probabilities obtained from the Markov-switching model,
"duration i" denotes the expected duration (in months) of each regime i.

Table 3bC - *Regime properties of MSH(3)-VECM(k-1)*

				duration		
MSH(M)-VECM(k-1)	p_{11}	p_{22}	p_{33}	regime 1	regime 2	regime 3
MSH(3)-VECM(5)	0.8692	0.9502	0.9091	7.65	20.07	11.00
MSH(3)-VECM(4)	0.8782	0.9456	0.8799	8.21	18.37	8.33
MSH(3)-VECM(3)	0.8762	0.9487	0.8916	8.08	19.50	9.23
MSH(3)-VECM(2)	0.8765	0.9598	0.9439	8.10	24.90	17.82
MSH(3)-VECM(1)	0.8723	0.9510	0.8991	7.83	20.40	9.91

p_{ii} denote the transition probabilities obtained from the Markov-switching model,
"duration regime i" denotes the expected duration (in months) of each regime i.

We may conclude that the specification with one lag is superior even in the two-regime model[27], and the comparison between three and two regimes is favorable to the three-regimes specification, in terms of all the information criteria. This conclusion is also confirmed by all the LR tests we have done. Comparing the information criteria reported in Table 1C and in Table 2C, it is difficult to choose between the models MSAH(3)-VECM(1) and MSIAH(3)-VECM(1). However, there is no difference in terms of the dating of the regimes, and also there is no difference with reference to all other important information related to the concept of weak exogeneity and volatility.[28] The results of the MSIAH(2)-VECM(1) and MSAH(3)-VECM(1) models are more informative with respect to the shift in the constant and the adjustment coefficients.

The dating of regimes for the MSAH(3)-VECM(1) version is presented in Figure 1C, and Table 4C reports the estimated coefficients.

[27] It is important to stress that adding more dynamics does not change the main information regarding the dating of the regimes and the statistical significance of the adjustment coefficients.

[28] Moreover, for the two models the underlying assumptions concerning autocorrelation and normality appear to be satisfied.

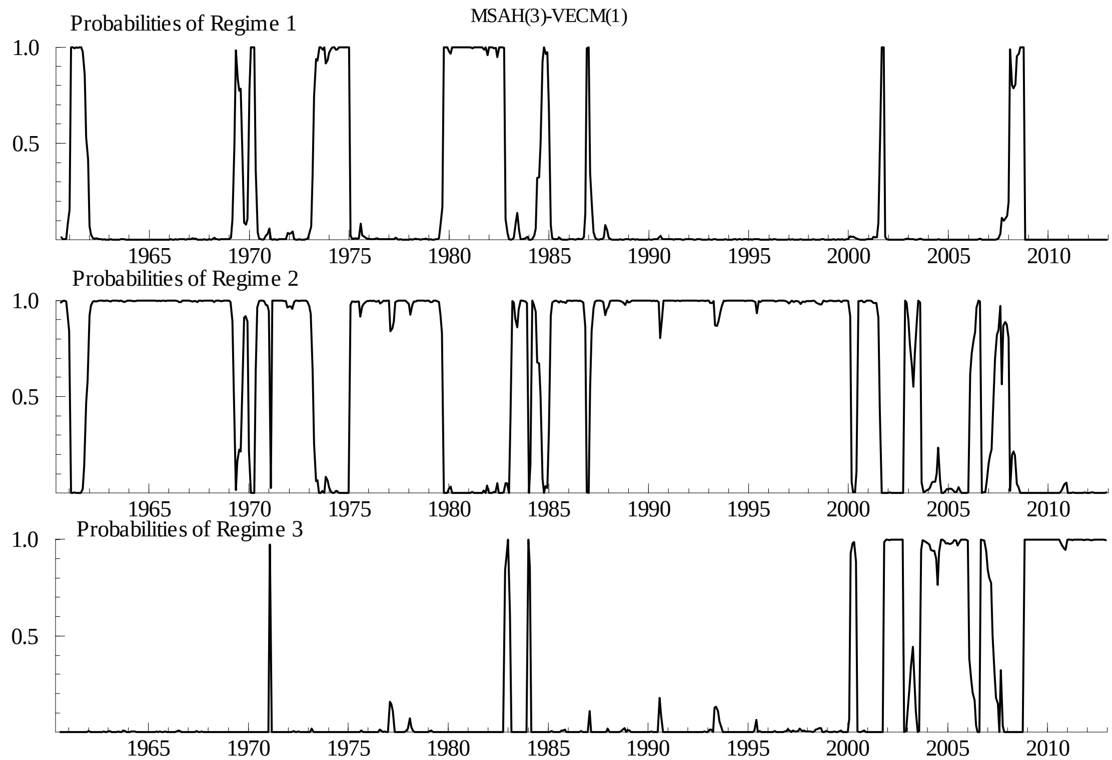


Figure 1C: Conditional (smoothed) probabilities of the three regimes obtained from MSAH(3)-VECM(1) for $\Delta\overline{R}_t$, $\Delta\overline{\pi}_t$, $\Delta\Theta_t$, and Δu_t with the equilibrium error $\beta' y_t = \overline{R}_t - 2.6\overline{\pi}_t + \Theta_t + 12.2 u_t$ restricted as exogenous variable.

Table 4C- *Estimated coefficients in the non-linear VECM(1)*

Regime 1	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	**0.017950**	0.003856	0.003423	**−0.002338**
$\Delta\overline{R}_{t-1}$	**0.321372**	0.053472	**−0.119070**	−0.000321
$\Delta\overline{\pi}_{t-1}$	0.033179	0.196502	−0.207637	−0.006186
$\Delta\Theta_{t-1}$	**0.754147**	**−0.281400**	**−0.239370**	−0.001810
Δu_{t-1}	**−11.82702**	−1.612646	−0.705350	**0.285474**
$\beta' y_{t-1}$	**−0.03326**	−0.004253	−0.001561	**−0.001627**
SE (Reg.1)	1.052867	0.397654	0.413859	0.033853
Regime 2	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	**0.017950**	0.003856	0.003423	**−0.002338**
$\Delta\overline{R}_{t-1}$	**0.472565**	0.117516	**−0.219777**	**−0.014840**
$\Delta\overline{\pi}_{t-1}$	**0.092320**	**0.31235**	**−0.181082**	−0.004065
$\Delta\Theta_{t-1}$	0.035742	−0.016915	0.571675	0.003124
Δu_{t-1}	**−1.634772**	0.011515	−0.237931	**−0.211028**
$\beta' y_{t-1}$	0.000528	0.003243	**−0.008594**	**−0.001213**
SE (Reg.2)	0.208758	0.255221	0.251750	0.026394
Regime 3	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	**0.017950**	0.003856	0.003423	**−0.002338**
$\Delta\overline{R}_{t-1}$	**0.663402**	**0.924057**	**−0.842949**	−0.025673
$\Delta\overline{\pi}_{t-1}$	−0.014583	**0.351489**	**−0.319073**	−0.005763
$\Delta\Theta_{t-1}$	0.008864	0.011150	**0.303993**	0.005797
Δu_{t-1}	0.111979	−1.540130	−1.204324	0.191255
$\beta' y_{t-1}$	**−0.002725**	**0.022789**	**−0.027218**	0.000571
SE (Reg.3)	0.051414	0.460859	0.624440	0.021234

Note. Bold characters mean rejection of the null hypothesis of zero coefficients at the 95% confidence level or higher.

Online Appendix D. A Two-State Markov-switching VECM

In this section we report the results of the two-state Markov-switching VECM [more precisely: MSIAH(2)-VECM(1)]. In this framework, all the tests support the non-linearity hypothesis: LR linearity test=907.1641, $\chi^2(34)$=[0.0000]**, $\chi^2(36)$=[0.0000]**. Moreover, the Davies (1987) upper bound test does not reject the non-linear model: DAVIES=[0.0000]**. Table 1C reports regime properties, and the matrix $\hat{P}$ is the estimated transition matrix. Table 2C presents the distinct set of the estimated parameters of the VECM in each regime endogenously separated by Markov-Switching methodology.

The two distinct regimes mostly differ with respect to a different adjustment to the equilibrium error and with respect to volatility.

The dating of regimes for the MSIAH(2)-VECM(1) is presented in Figure 1D. Figure 2D shows that the most remarkable periods prevailing in regime 1 are clearly identified as NBER recession periods also when we consider only two-state Markov-switching.

$$\hat{P} = \begin{pmatrix} \hat{p}_{11} & \hat{p}_{12} \\ \hat{p}_{21} & \hat{p}_{22} \end{pmatrix} = \begin{pmatrix} 0.86 & 0.13 \\ 0.03 & 0.97 \end{pmatrix}$$

Table 1D - *Regime properties*

	nObs	Prob	Duration
Regime 1	108.0	0.1706	6.98
Regime 2	521.0	0.8294	33.93

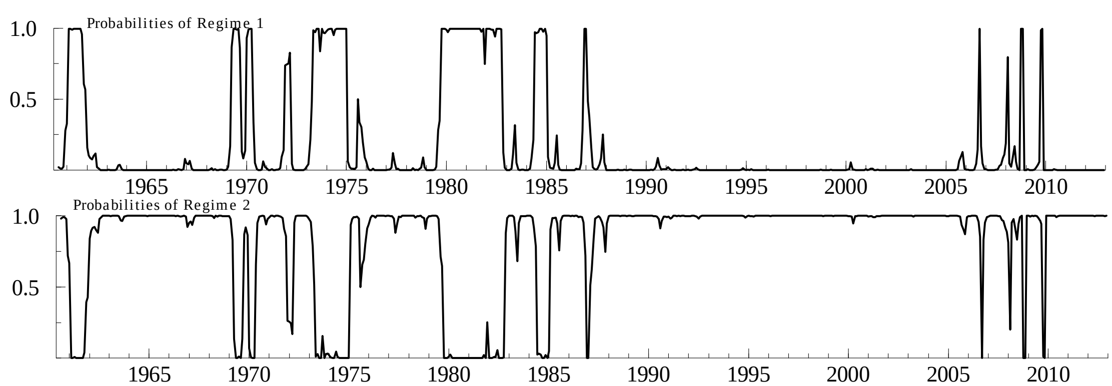


Figure 1D: Conditional (smoothed) probabilities of the two regimes obtained from MSIAH(2)-VECM(1) for $\Delta\overline{R}_t$, $\Delta\overline{\pi}_t$, $\Delta\Theta_t$, and Δu_t with the equilibrium error $\beta' y_t = \overline{R}_t - 2.6\overline{\pi}_t + \Theta_t + 12.2u_t$ restricted as exogenous variable.

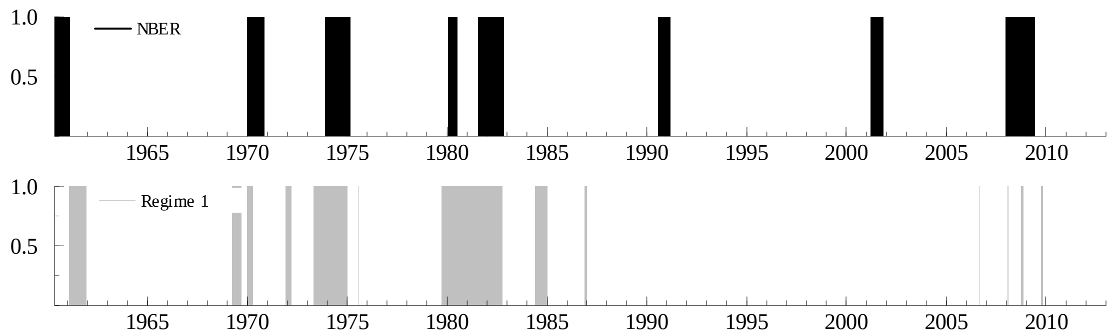


Figure 2D: NBER recession dates (shadowed black areas) compared with smoothed probabilities of Regime 1 (shadowed grey areas).

Table 2D - Estimated coefficients in the non linear VECM(1)

Regime 1	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	**0.618940**	−0.116170	0.055603	**0.036655**
$\Delta\overline{R}_{t-1}$	**0.304398**	0.043001	**−0.116891**	−0.000576
$\Delta\overline{\pi}_{t-1}$	0.199760	**0.375792**	−0.101656	−0.006421
$\Delta\Theta_{t-1}$	**0.784600**	**−0.489908**	**0.439680**	−0.005025
Δu_{t-1}	**−11.61071**	−1.537988	−1.277024	**0.214426**
$\beta' y_{t-1}$	**−0.022735**	0.003336	−0.003136	**−0.001237**
SE (Reg.1)	1.020657	0.474899	0.326292	0.031569
Regime 2	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	0.009180	−0.042348	**0.319242**	**0.013478**
$\Delta\overline{R}_{t-1}$	**0.520426**	**0.182668**	**−0.272291**	**−0.021101**
$\Delta\overline{\pi}_{t-1}$	0.027547	**0.291142**	**−0.312431**	−0.004958
$\Delta\Theta_{t-1}$	0.024176	0.017141	**0.422738**	0.004043
Δu_{t-1}	**−1.393898**	−0.107293	−0.186801	**−0.110677**
$\beta' y_{t-1}$	−0.000292	0.002110	**−0.012857**	**−0.000651**
SE (Reg.2)	0.180859	0.283529	0.389653	0.026396

Note. Bold characters mean rejection of the null hypothesis of zero coefficients at the 95% confidence level or higher.

Online APPENDIX E: Analysis with M1

In this section we report the results of the cointegration analysis and Markov-Switching VECM where M1 is used instead of M2. Following the approach by Carlson and Schwarz (1999) and Mehra (1997) we make use of a dummy variable that introduces a linear shift in the monetary aggregate from 1990:1 to 1994:3. Also we make use of two step dummies for 1980:11-12, and for 1981:2, to account for a one-time shift in the variable in those years.

Table 1E reports the results of the cointegration test.

Table 1E: Test for cointegrating rank

Rank	0	1	2	3
Trace test [Prob]	57.51[0.022]*	24.51[0.435]	9.28[0.712]	3.05[0.580]
Max test [Prob]	33.00[0.010]*	15.23[0.369]	6.23[0.761]	3.05[0.579]
Trace(T-nm) [Prob]	55.32[0.037]*	23.58[0.495]	8.93[0.744]	2.94[0.601]
Max(T-nm) [Prob]	31.74[0.016]*	14.65[0.418]	5.99[0.786]	2.94[0.600]

Note. The trace test and the max test are the log-likelihood ratio tests (LR), which are based on the four eigenvalues (0.051, 0.024, 0.010 and 0.005). The VAR tested for cointegration is a VAR(6) with an intercept in the cointegrating vector. The row denoted as rank reports the number of cointegrating vectors, and [prob] indicates the p-value computed from critical values by Doornik (1998). The last two rows report small sample correction.

The restricted relationship with $\beta_\Theta = 1$, which is not rejected [Likelihood Ratio test: $\chi^2(3) = 7.7586[0.0513]$], is the following.

$$\beta' y_t = \bar{R}_t - 1.7\bar{\pi}_t + (\Theta_t - \bar{\pi}_t) + 12.7u_t - 23.1 \tag{15}$$

Figure 1E graphs the equilibrium error.

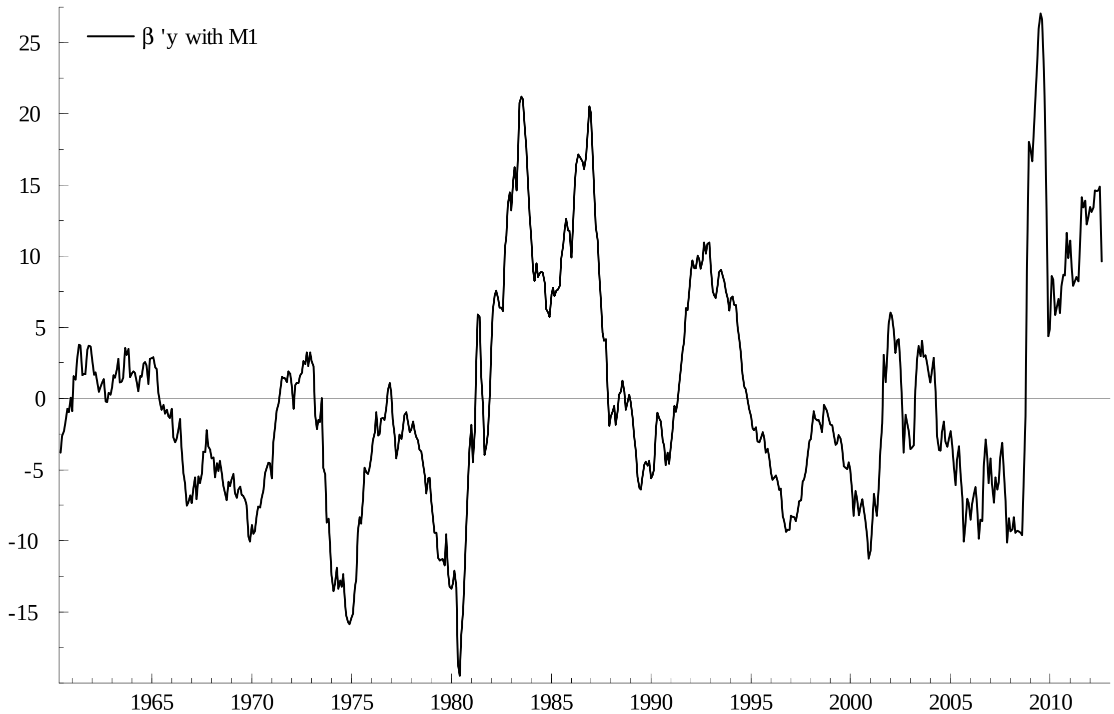


Figure 1E: Equilibrium error $\beta' y_t = \bar{R}_t - 1.7\bar{\pi}_t + (\Theta_t - \bar{\pi}_t) + 12.7u_t - 23.1$; where Θ_t is the rate of growth of $M1$.

Table 3E presents the results of the MSIAH(3)-VECM(1), and Figure 2E graphs the probabilities of the three regimes.

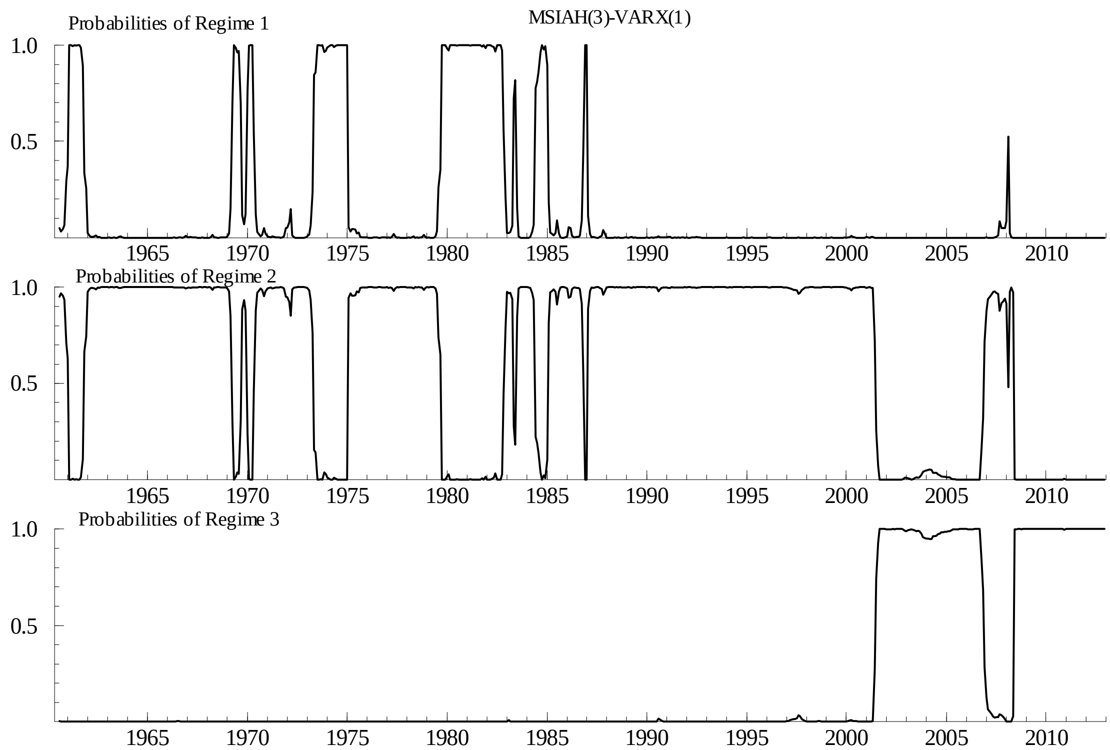


Figure 2E: Conditional (smoothed) probabilities of the three regimes obtained from MSIAH(3)-VECM(1) for $\Delta\overline{R}_t$, $\Delta\overline{\pi}_t$, $\Delta\Theta_t$, and Δu_t with the equilibrium error $\beta' y_t = \bar{R}_t - 1.7\bar{\pi}_t + (\Theta_t - \bar{\pi}_t) + 12.7u_t - 23.1$ restricted as exogenous variable, and where Θ_t is the rate of growth of $M1$.

Table 3E- *Estimated coefficients in the non-linear VECM(1)*

Regime 1	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	**0.725476**	0.162575	−0.004009	**0.032877**
$\Delta\overline{R}_{t-1}$	**0.301578**	**0.071922**	**−0.218912**	−0.000511
$\Delta\overline{\pi}_{t-1}$	0.147334	0.175229	−0.364857	−0.005678
$\Delta\Theta_{t-1}$	**0.799274**	−0.058540	**0.320868**	−0.005406
Δu_{t-1}	**−11.046582**	−1.643016	−1.291753	0.177589
$\beta' y_{t-1}$	**−0.029573**	−0.008894	0.001438	**−0.001070**
SE (Reg.1)	0.954820	0.396272	0.656355	0.031363
Regime 2	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	−0.006011	−0.065671	**0.184800**	**0.015080**
$\Delta\overline{R}_{t-1}$	**0.494688**	**0.151708**	**−0.270768**	**−0.017348**
$\Delta\overline{\pi}_{t-1}$	**0.087200**	**0.295509**	−0.138790	−0.003439
$\Delta\Theta_{t-1}$	0.026916	0.006289	**0.339859**	0.001826
Δu_{t-1}	**−1.572895**	0.191250	−0.071964	**−0.218376**
$\beta' y_{t-1}$	0.000122	0.003610	**−0.008027**	**−0.000852**
SE (Reg.2)	0.204349	0.262511	0.556534	0.026696
Regime 3	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	**0.017950**	0.003856	0.003423	0.002780
$\Delta\overline{R}_{t-1}$	**0.663402**	**0.924057**	**−0.842949**	**−0.049276**
$\Delta\overline{\pi}_{t-1}$	−0.014583	**0.351489**	**−0.319073**	−0.000381
$\Delta\Theta_{t-1}$	0.008864	0.011150	**0.303993**	**0.004102**
Δu_{t-1}	0.111979	−1.540130	−1.204324	**0.219606**
$\beta' y_{t-1}$	**−0.002725**	**0.022789**	**−0.027218**	−0.000050
SE (Reg.3)	0.051414	0.460859	0.624440	0.020191

Note. Bold characters mean rejection of the null hypothesis of zero coefficients at the 95% confidence level or higher.

Figure 3F, 4F and 5F show that also with M1 regime 1 tends to coincide with NBER recessions, regime 2 with NBER expansions, and regime 3 with negative interest rate periods. Figure 6F graphs the money supply growth rate of M1 minus the inflation rate in the dashed line, and the equilibrium error with M1.

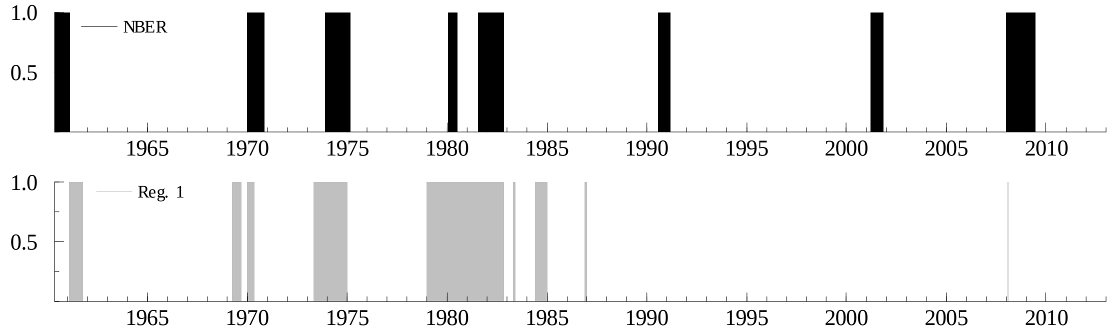


Figure 3E: NBER recession dates (shadowed black areas) compared with smoothed probabilities of regime 1 (shadowed grey areas)

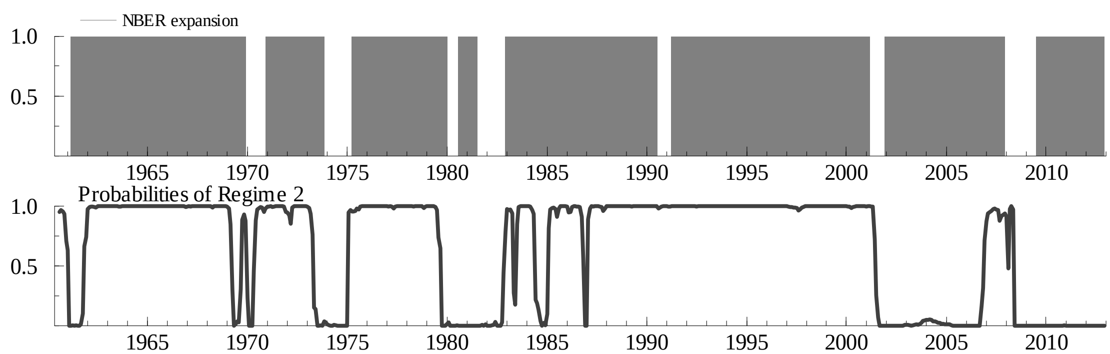


Figure 4E: NBER expansions dates (shadowed grey areas) compared with smoothed probabilities of regime 2

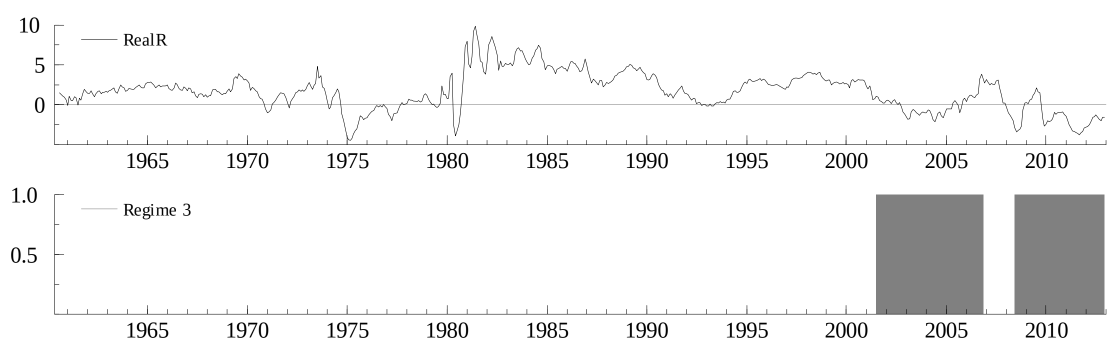


Figure 5E: Regime 3 compared with the real interest rate.

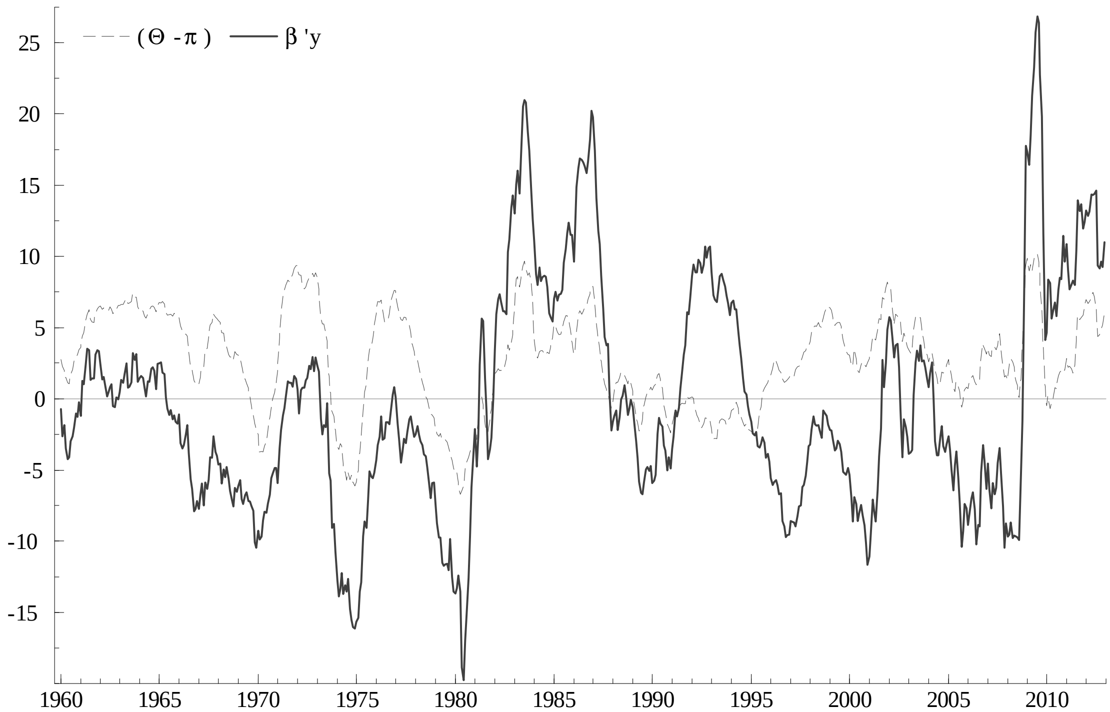


Figure 6E: Growth rates of real balances (M1) and the VECM Equilibrium Error Term

Online APPENDIX F: Analysis with the 3-Month Treasury Bill rate

We present the same analysis using the 3-Month Treasury Bill rate[29] (R3m) instead of the Federal Fund rate (FFR).

In this case, we choose a VAR(7) since it does not present autocorrelation of the 1st order [30]. Table 1F presents the tests for cointegrating rank.

[29] Source: Board of Governors of the Federal Reserve System (the series is not seasonally adjusted).

[30] Testing for AR(1) vector error autocorrelation produces the following result: $\chi^2(16) = 14.495[0.5619]$. Testing for AR(1) in single-equation produces the following results:
in equation for R: $F(1,599) = 1.0529[0.3052]$; in the equation for u : $F(1,599) = 0.0087453[0.9255]$;in the equation for Θ: $F(1,599) = 8.0037e-005[0.9929]$; and in equation for π: $F(1,599) = 0.99450[0.3190]$. In square brackets we report the *p-value*.

Table 1F: Test for cointegrating rank

Rank	0	1	2	3
Trace test [Prob]	75.88[0.000]**	28.96[0.203]	13.24[0.353]	4.26[0.387]
Max test [Prob]	46.92[0.000]**	15.72[0.330]	8.98[0.448]	4.26[0.386]
Trace(T-nm) [Prob]	72.50[0.000]**	27.68[0.259]	12.65[0.401]	4.07[0.414]
Max(T-nm) [Prob]	44.83[0.000]**	15.02[0.386]	8.58[0.492]	4.07[0.413]

Note. The trace test and the max test are the log-likelihood ratio tests (LR), which are based on the four eigenvalues (0.072, 0.025, 0.014 and 0.007). The VAR tested for cointegration is a VAR(7) with an intercept in the cointegrating vector. The row denoted as rank reports the number of cointegrating vectors, and [prob] indicates the p-value computed from critical values by Doornik (1998). The last two rows report small sample correction.

Table 2F reports the cointegrated and loading coefficients. In this contest, we test if Θ is not a relevant variable for cointegration, but the LR test on $\beta_\Theta = 0$ strongly rejects this hypothesis: $\chi^2(1) = 6.829[0.0090]^{**}$. In addition, as the coefficient α_π is not significantly different from zero, we also test the restriction[31] $\alpha_\pi = 0$, which is not rejected.[32] Imposing the restriction $\beta_\Theta = 1$, which is not rejected ($\chi^2(2) = 0.85095[0.6535]$), the equilibrium relation could be expressed making explicit the rate of growth of real balances $(\Theta_t - \bar{\pi}_t)$. All these results are presented in Table 2Fa.

Table 2F: Cointegrated coefficients and loading coefficients

	Cointegrated coefficients	*Loading coefficients*
$\bar{R}_t$	1	$\alpha_R = -0.012$ (0.004)
$\bar{\pi}_t$	-2.146 (0.247)	$\alpha_\pi = 0.003$ (0.003)
Θ_t	0.744 (0.235)	$\alpha_\Theta = -0.013$ (0.004)
u_t	9.170 (2.402)	$\alpha_u = -0.001$ (0.0003)
$Const.$	-17.965 (4.309)	

Note. The standard errors are presented in the round parentheses

[31] Table 2Ga also reports the tests of weak exogeneity on all the variables.
[32] This means that π is a weak exogenous variable.

Table 2Fa:. Multivariate cointegration analysis

	Cointegrated coefficients	Loading coefficients
$\bar{R}$	1	$\alpha_R = -0.009$ (0.0033)
$\bar{\pi}_t$	-2.372 (0.285)	$\alpha_\pi = \ 0$
Θ_t	1	$\alpha_\Theta = -0.011$ (0.0032)
u_t	11.209 (2.871)	$\alpha_u = -0.001$ (0.0002)
$Const.$	-22.448 (5.023)	
	Test of weak exogeneity	**LR test of restrictions:**
Restriction:	$\alpha_R = 0$	$\chi^2(1) = 6.9595[0.0083]^{**}$
Restriction:	$\alpha_\pi = 0$	$\chi^2(1) = 0.5085[0.4758]$
Restriction:	$\alpha_\Theta = 0$	$\chi^2(1) = 7.6639[0.0056]^{**}$
Restriction:	$\alpha_u = 0$	$\chi^2(1) = 11.104[0.0009]^{**}$

Note.The standard errors are presented in the round parentheses, while the p-values are reported in the square brackets.

In Figure F1 we report the results of the chosen three-state Markov-switching VECM, more precisely the MSIH(3)-VECM(1).

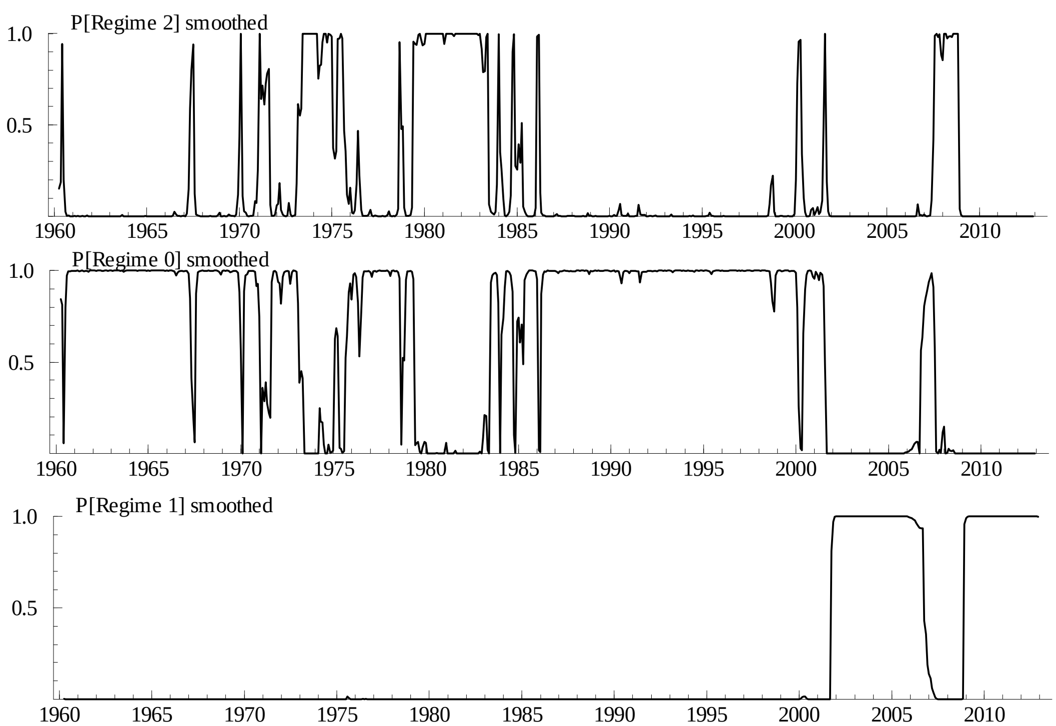


Figure F1: Conditional (smoothed) probabilities of the three regimes obtained from the MSIH(3)-VECM(2) for $\Delta\overline{R}_t$, $\Delta\overline{\pi}_t$, $\Delta\Theta_t$, and Δu_t with the equilibrium error $\beta' y_t = \overline{R}_t - 2.4\overline{\pi}_t + \Theta_t + 11.2u_t$ restricted as exogenous variable. $\overline{R}_t$ is the 3-Month Treasury Bill rate.

In Table 3F we report the estimated coefficients of the chosen non linear MSIH(3)-VECM(2). Moreover, $\widehat{P}$ is the estimated matrix of transition probabilities.

$$\widehat{P} = \begin{pmatrix} 0.962 & 0.011 & 0.106 \\ 0.000 & 0.989 & 0.016 \\ 0.038 & 0.000 & 0.878 \end{pmatrix}$$

Table 3F - Estimated coefficients in the non linear VECM(2)

	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.(\mathrm{Re}\,g1)$	0.124584	0.121654	0.147848	**0.030370**
$Const.(\mathrm{Re}\,g2)$	0.046325	−0.100981	0.098238	**0.027473**
$Const.(\mathrm{Re}\,g3)$	**0.135227**	**−0.505664**	**0.712339**	−0.015092
$\Delta\overline{R}_{t-1}$	**0.379653**	**0.086800**	**−0.132962**	−0.001791
$\Delta\overline{R}_{t-2}$	−0.052027	**0.081214**	−0.036808	**−0.006144**
$\Delta\overline{\pi}_{t-1}$	0.031693	**0.300816**	**−0.168840**	−0.002797
$\Delta\overline{\pi}_{t-2}$	−0.015280	−0.045215	−0.055333	−0.000664
$\Delta\Theta_{t-1}$	−0.015702	−0.043603	**0.536127**	0.003830
$\Delta\Theta_{t-2}$	0.018661	−0.017179	−0.034466	−0.000406
Δu_{t-1}	**−1.048250**	−0.004209	0.081080	**−0.082310**
Δu_{t-2}	**−0.709059**	−0.551624	**1.219010**	**0.137594**
$\beta' y_{t-1}(\mathrm{Re}\,g1)$	−0.007053	−0.008009	−0.006875	**−0.000778**
$\beta' y_{t-1}(\mathrm{Re}\,g2)$	−0.002032	0.004980	−0.004070	**−0.001348**
$\beta' y_{t-1}(\mathrm{Re}\,g3)$	**−0.005281**	**0.022340**	**−0.031483**	0.000534

Note. Bold characters mean rejection of the null hypothesis of zero coefficients at the 95% confidence level or higher. The estimated model is the MSIH(3)-VECM(2) for $\Delta\overline{R}_t$, $\Delta\overline{\pi}_t$, $\Delta\Theta_t$, and Δu_t with the equilibrium error $\beta' y_t = \overline{R}_t - 2.4\overline{\pi}_t + \Theta_t + 11.2u_t$ restricted as exogenous variable. $\overline{R}_t$ is the 3-Month Treasury Bill rate.